Neutrinos in the Grimus–Neufeld model

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Outline

Motivation

- BSM physics already:
  - neutrinos have mass and mix...
    - but what is the exact mechanism?
- Unknown BSM physics: More scalars?

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- BSM physics already:
  - neutrinos have mass and mix...
    - but what is the exact mechanism?
- Unknown BSM physics: More scalars?
- Being general but minimal:
  - 2HDM + 1 Seesaw neutrino + $\rightarrow$ Grimus–Neufeld model.
    - Incorporates masses and mixings at one loop.
- Seesaw models induce LFV.

Dirac or Majorana

- Dirac and Majorana spinors in chiral basis

$$\psi = \begin{pmatrix} e \\ E^\dagger \end{pmatrix}, \vartheta = \begin{pmatrix} \nu \\ \nu^\dagger \end{pmatrix}, \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

e, ν – LH, $E^\dagger, \nu^\dagger$ – RH

  - Majorana has RH $= \overline{\text{LH}} \Rightarrow$ 2 d.o.f.s instead of 4.

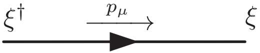
- Dirac propagator

$$\langle \psi \bar{\psi} \rangle = i \frac{\gamma^\mu p_\mu + m}{p^2 - m^2},$$

can be decomposed into $\sim \sigma^\mu$ or $\sim \bar{\sigma}^\mu$ as chirality preserving and $\sim m$ as chirality violating terms.

Diagramatic representation

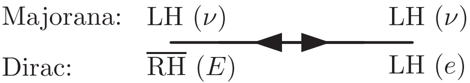
- Arrow shows the direction of left chirality propagation (see [Dreiner, Haber, Martin '10]):



$$\sim \sigma^\mu p_\mu \text{ or } -\bar{\sigma}^\mu p_\mu$$

$\xi = e, E, \nu$ are LH Weyl spinors.

- Propagators that $\sim m$ differ for Dirac and Majorana:



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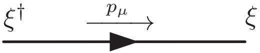

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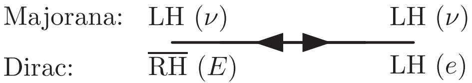

Dirac type connects $\overline{\text{RH}}$ with LH
Majorana type connects LH with LH.

- Consider propagation from left to right:
  - RH ($\overline{\text{LH}}$) antineutrino becomes LH neutrino.
  - RH electron becomes LH electron

propagator types

$\xi = e, E, \nu$

$\xi^\dagger$ $\xrightarrow{p_\mu}$ ξ

$$\frac{i\sigma p}{p^2 - m_\xi^2} \text{ or } -\frac{i\bar{\sigma} p}{p^2 - m_\xi^2}$$

Majorana: LH (ν) ... LH (ν)

Dirac: $\overline{\mathrm{RH}}$ (E) ... LH (e)

$$\frac{i m_\xi}{p^2 - m_\xi^2}$$

Majorana: $\overline{\mathrm{LH}}$ ($\nu^\dagger$) ... $\overline{\mathrm{LH}}$ ($\nu^\dagger$)

Dirac: $\overline{\mathrm{LH}}$ ($e^\dagger$) ... RH ($E^\dagger$)

$$\frac{i m_\xi^\dagger}{p^2 - m_\xi^2}$$

Seesaw

- Why SM neutrinos do not have a mass:
  - Majorana mass term violates gauge invariance explicitly
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    - which needs independent RH component

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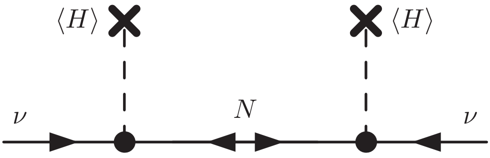
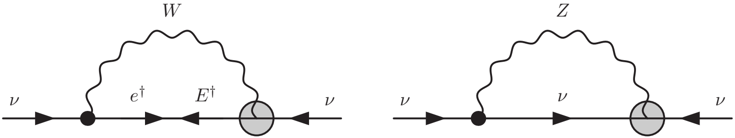
  - Majorana mass term violates gauge invariance explicitly
  - EWSB generates only Dirac type mass,
    - which needs independent RH component
- Sesaw mechanism:
  - Introduces independent RH component $N^\dagger$.
    - Allows EWSB generated Dirac mass
    - RH dof is singlet $\Rightarrow$ Majorana mass M for RH component allowed.
  - Generates effective $\sim 1/M$ Majorana masses in EWSB phase:



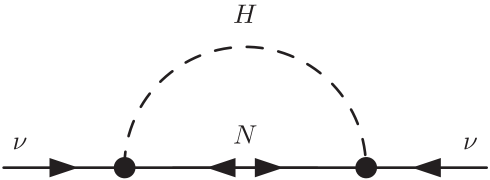
Seesaw with loops: radiative mass?

- Radiative mass: mass, generated via loops
- Why SM neutrinos do not have radiative mass (one loop):



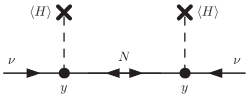
These diagrams are impossible in the SM

- Include particle N, having a Majorana mass (connects LH with LH):



No radiative mass in SM+seesaw

- O(5) operator representation from seesaw:



- Effective mass term, from integrating out heavy N:

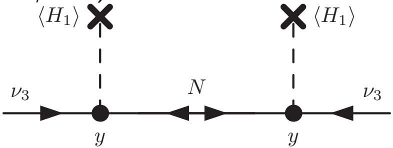
$$\mathscr{L} = \frac{1}{\sqrt{2}} y \nu N v + \frac{1}{2} M N^2 \rightarrow \frac{v^2 y^2}{2M} \nu \nu$$

- y determines the coupling of ν to scalar **and** the mass term.

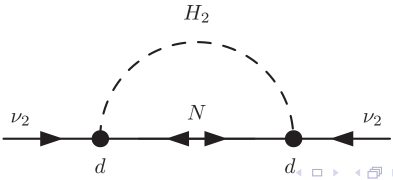
  $\Rightarrow$ one heavy N leads to one 1/M neutrino mass
  $\Rightarrow$ loop corrections contributes to the seesaw mass,
  but does not induce more massive neutrino states..
  $\Rightarrow$ Needs more then 1 d.o.f at high scale

Grimus-Neufeld model

- Take another scalar doublet $\Rightarrow$ 2HDM+ 1 heavy N [G-N '89]. Can we fit masses?
- One neutrino, ν_3, gets seesawed with $\langle H_1 \rangle$(Higgs basis, where $\langle H_1 \rangle = v/\sqrt{2}$, $\langle H_2 \rangle = 0$):



- another, ν_2, gets mass radiatively with H_2:



GN model

- ν_1 stays masless at one loop.
- 2HDM gives us 2 general complex 3-vectors as Yukawa couplings Y_ν^1 and Y_ν^2 in flavour and the Higgs basis:

$$\mathscr{L} = -Y^1_{\nu_i}\ell_i H_1 N - Y^2_{\nu_i}\ell_i H_2 N + H.c., i = e, \mu, \tau$$

$$H_1 = \begin{pmatrix} G_W^+ \\ \frac{1}{\sqrt{2}}(v + h + iG_Z) \end{pmatrix}, H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix},$$

GN model

- 4×4 mixing matrix, relates flavor basis to mass eigenstate basis:

$$\nu^F = U_\nu \nu^M = V_{PMNS} U_{\text{seesaw}} \nu^M$$

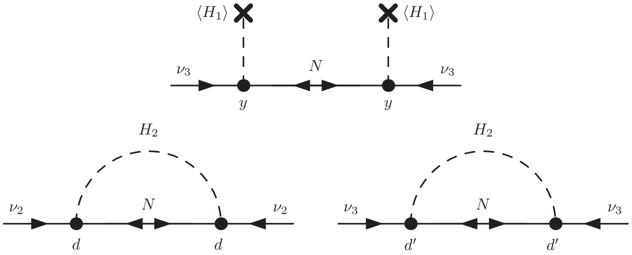
- 3×3 block of U_ν is approximately Unitary and should correspond PMNS from experiment.
- V_{PMNS} is exacly unitary 3×3, which we use to pick the basis:

$$Y_\nu^1 V_{PMNS} = (0, 0, y)\ ,\ Y_\nu^2 V_{PMNS} = (0, d, d')$$

which is approximate 1 loop mass eigenstate basis (next slide)

GN model

$$Y_\nu^1 V_{PMNS} = (0,0,y)\ ,\ Y_\nu^2 V_{PMNS} = (0,d,d')$$



- Task: take PMNS, Δm_{12}^2, and Δm_{13}^2 from experiment and relate them to d, d', y at one loop.

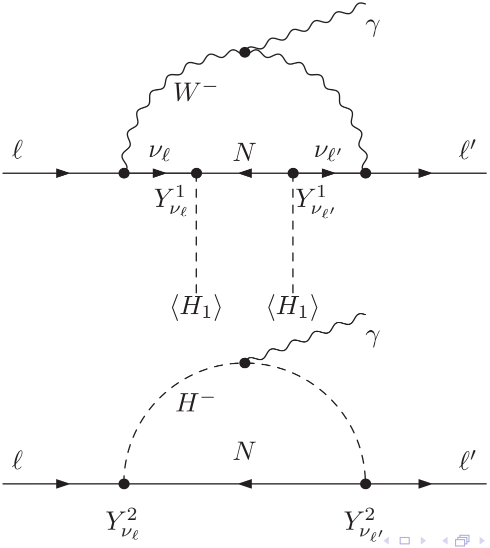
Why bother doing it?

- neutrino couplings Y_ν^1 and Y_ν^2 are fully determined by y, d, d' and V_{PMNS}
  - y, d, d' depend on $\sqrt{\Delta m_{21}^2}$, $\sqrt{\Delta m_{31}^2}$, Higgs masses and mixings
- One can look at processes, where Y_ν^1 and Y_ν^2 appears:
  - $\ell \to \ell' \gamma$,
  - anomalous magnetic moment
  - $H^- \to \ell^- \nu$
  - ...
- Then one can combine these with neutrino data

$\Rightarrow$ they interplay with electron Yukawas

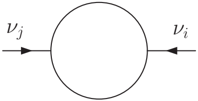
$\Rightarrow$ could also restrict the scalar sector

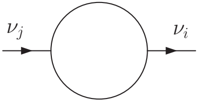
Example for $\ell \to \ell'\gamma$



1 Loop

- Calculating the effective two point functions, we get corrections:

$i\Gamma_{ij} =$ 

$i\sigma p \Sigma_{ij} =$ 

- Expanding in loops :

$$\Gamma^{[\leq 1]} = \Gamma^{[0]} + \Gamma^{[1]},\ \Gamma^{[0]}_{33} = -m_3 \approx -\frac{y^2 v^2}{2M},\ \Gamma^{[0]}_{44} = -m_4 \approx -M$$

- Tree and loop effective mass-like two pt functions look like:

$$\Gamma^{[0]} = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix},\ \Gamma^{[1]} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix}$$

Pole masses

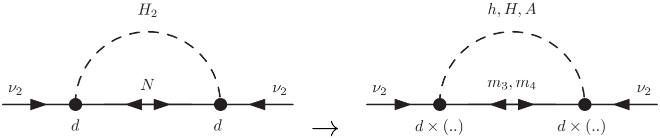
- One loop approximation give four pole masses:

$$\mu_1 = 0 \qquad \mu_3 = m_3 - \Gamma^{[1]}_{33} - m_3 \Sigma^{[1]}_{33}$$
$$\mu_2 = -\Gamma^{[1]}_{22} \qquad \mu_4 = m_4 - \Gamma^{[1]}_{44} - m_4 \Sigma^{[1]}_{44}$$

- μ_2 is radiatively generated mass, μ_3 is corrected light seesaw mass, and heavy $\mu_4 \sim M$ at one loop.
- The most of the 2pt functions need to be defined in the renormalization scheme, except for $\Gamma^{[1]}_{22}$
  - there is no counterterm available, since tree level mass is zero.

Radiative mass

- The result for $\Gamma_{22}^{[1]}$:



- or:

$$\mu_2 = -\Gamma_{22} = -\frac{d^2}{32\pi^2 (m_3 + m_4)} \times$$
$$\times \Big(m_3^2 \left[B_0 (0, m_3^2, m_A^2) - s_{\beta-\alpha}^2 B_0 (0, m_3^2, m_H^2) - c_{\beta-\alpha}^2 B_0 (0, m_3^2, m_h^2) \right]$$
$$- m_4^2 \left[B_0 (0, m_4^2, m_A^2) - s_{\beta-\alpha}^2 B_0 (0, m_4^2, m_H^2) - c_{\beta-\alpha}^2 B_0 (0, m_4^2, m_h^2) \right] \Big) .$$

- Finite and gauge invariant without the need of any UV subtraction.

Radiative mass

- One loop approximation gives one of the pole mass relation:

$$\Gamma_{22} \equiv d^2 \tilde{\Gamma}_{22} \Rightarrow \; d^2 = -m_2 / \tilde{\Gamma}_{22}$$

  - The functional dependency:

$$d^2 = f\left(m_h, m_A, m_H, \alpha - \beta, m_2, m_3, m_4\right)$$

$\Rightarrow$ relates $m_h, m_A, c_{\alpha-\beta}$ to neutrino parameters

  - note: relation breaks down, when $m_A = m_H$ and $c_{\alpha-\beta} = 0$

- For simplicity, assuming NH:

$$m_2 = \sqrt{\Delta m^2_{21}}\,, \; m_3 = \sqrt{\Delta m^2_{31}}$$

$\Rightarrow$ we related d with $\sqrt{\Delta m^2_{21}}$

- Let us go on an use the other mass.

Corrections for seesaw mass

- The one loop seesaw mass:

$$\mu_3 = m_3 - \Gamma_{33}^{[1]} - m_3 \Sigma_{33}^{[1]}$$

  - loop functions are not finite $\Rightarrow$ needs renormalization scheme.
- In the OS, we can fix a relation to hold at one loop

$$y = \sqrt{\Delta m_{13}} \cdot m_4 / 2v$$

- Determine counterterms, check gauge invariance...
- d' then should be determined from other renormalization condition...

$\Rightarrow$ in general, need to renormalize the full model

Some other options?

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- We try to use FlexibleSUSY:
  - FS calculates pole masses from couplings
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  - Need relations between FS inputs and masses
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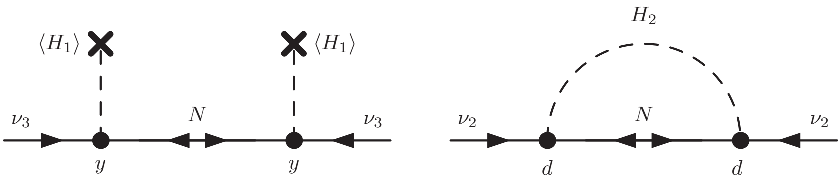
    - then we can parametrize mass shift with d' or..
  - try to use Grimus-Lavoura approximation for both masses
  - First check - we should get the same masses as output from the input of y, d, d' and V_{PMNS}
    - also ongoing research to implement in FS...

Grimus-Lavoura approximation

Inspecting the similarity



- Seesaw and loop are treated as the same order [Grimus, Lavoura '02]
  - $\Rightarrow$ There are no tree level masses for light neutrinos
  - $\Rightarrow$ there are no possible counterterms for UV subtraction of effective light mass matrix at one loop
  - $\Rightarrow$ loop corrections to light neutrino masses must be gauge invariant and finite

Grimus-Lavoura approximation

- Modify loop ordering in perturbative calculations from:

$$\Gamma^{[0]} = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \Gamma^{[1]} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix}$$

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- where we set:

$$m_3 \approx \frac{y^2 v^2}{2M} = O\,(1 \text{ loop})$$

- Pole masses:

$$\mu_2 \mu_3 = \Gamma^{[1]}_{22} \Gamma^{[1]}_{33} - \left(\Gamma^{[1]}_{23}\right)^2$$

$$\mu_2 + \mu_3 = -\Gamma^{[1]}_{22} - \Gamma^{[1]}_{33}$$

$$\Gamma^{[1]}_{22} \sim d^2\,,\ \Gamma^{[1]}_{23} \sim dd'\,(..) + yd'\,(..)\,,\ \Gamma^{[1]}_{33} \sim y^2\,(..) + (d')^2\,(...)$$

Pole masses

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- 2pt funtions $i\sigma p\Sigma$ do not enter at this order.
- No need for counterterms - finite and gauge invariant on themselves [Grimus, Lavoura '02]
    - the gauge dependent parts are multiplied by m_3,
    - but zeroth order m_3 is set to zero.
- Relates d, d' and y to Δm^2_{21} and Δm^2_{31}.
- The mixing terms are included in the approximation.

Some progress and open questions

- We managed to make FS working with GN model
- Some initial checks are being done:
  - GL approximated PMNS and masses seems to be reasonably reproduced with FS
    - the checks are not finalized yet..
- Difficulties:
  - Hierarchy problem:
    - in $\overline{\text{MS}}$ Higgs mass correction $\sim M_{\text{seesaw}}$

    $\Rightarrow$ it limits (roughly) $M_{\text{seesaw}} < 10^{4(5)}$ GeV from perturbativity
  - huge numerical cancellations

  $\Rightarrow$ need functions with many digits precision

Summary

- GN - a model that can incorporate the measured neutrino data
- 2HDM +1N – only 1 heavy scale
  - enough to have 2 mass differences and mixings at one loop
- Relatively small number of parameters in neutrino Yukawas. $(V_{PMNS}, y, d, d', M_{\text{seesaw}})$
  - Relates neutrino sector with scalar sector.
  - Contribute to LFV observables.
- Future goal: restrict parameters of scalar and Yukawa sector, including the neutrino data and the observables such as a_μ, $\mu \to e\gamma$, etc.