

## Modeling two-nucleon knock-out in neutrino-nucleus scattering

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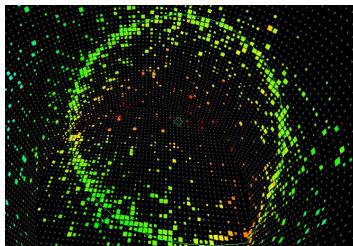
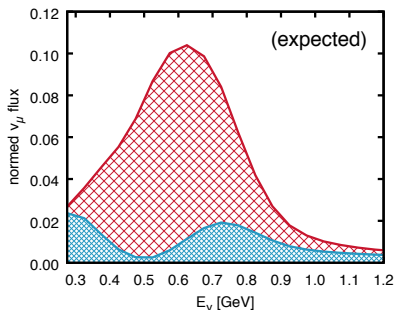


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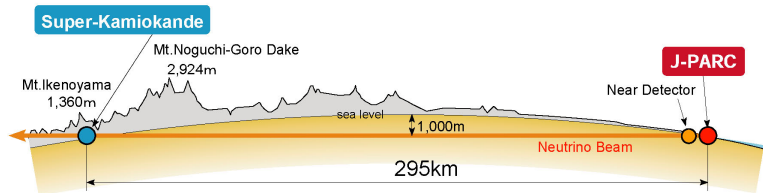
# Neutrino oscillation experiments



$$P_{2f}(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$



$$E_\nu^{\text{rec}} = \frac{2(M_n - E_B)E_\mu - (E_B^2 - 2M_n E_B + m_\mu^2)}{2[M_n - E_B - E_\mu + |\vec{k}_\mu| \cos \theta_\mu]}$$



# Detected rate of $\nu_\alpha$ events

$$R_{\nu_\alpha} \sim \Phi_{\nu_\mu}(E_\nu) \times P_{\nu_\mu \rightarrow \nu_\alpha}(\{\Theta\}, E_\nu) \times \sigma_{\nu_\alpha}(E_\nu) \times \epsilon_{\text{det.}}$$

Event rate

Incoming flux

Oscillation probability

Cross section

Efficiency

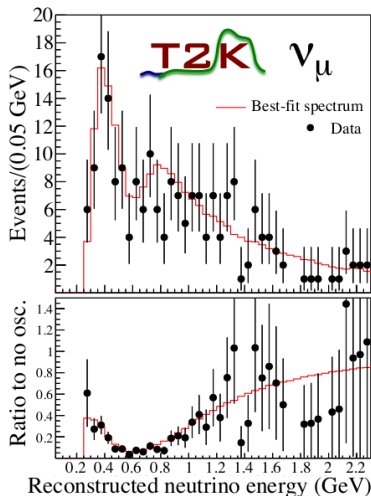
Knowledge of neutrino-nucleus **cross sections**:

- allows to **reconstruct neutrino energy** from the detected **final states**,
- is the **crucial uncertainty** in **oscillation analyses**,

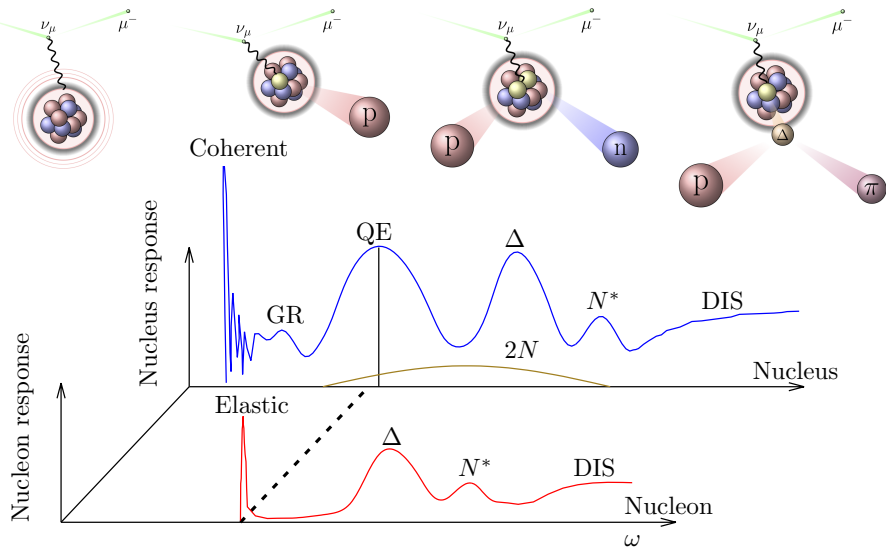
but...

- is an **advanced computational problem**,
- current **precision** is not exceeding **20%**,
- **constraints** from **ND** are **not enough**.

K. Abe et al., Phys.Rev.Lett. 121 (2018) 171802 (edited)

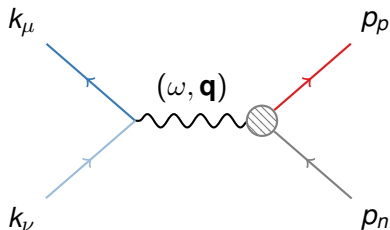


# Nuclear response



T. Van Cuyck

# Dimensionality of the problem



any binary scattering with on-shell particles

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4 four-vectors = **16 variables**

- 4 : on-shell relations
- 4 : 4-mom. conservation
- 3 : nucleon rest frame
- 2 : neutrino along  $\hat{z}$

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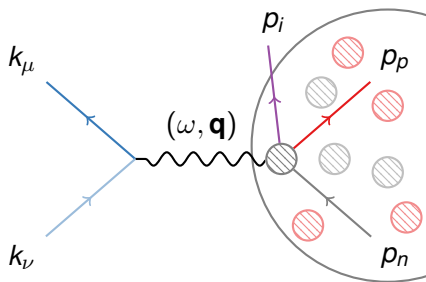
**3 independent variables**

→ we can fix incoming energy ( $E_\nu$ )

→ the cross section is rotationally invariant ( $\phi_\mu$ )

→ the final formula is 1-dimensional, e.g.  $d\sigma/dq^2$

# Dimensionality of the problem



*scatterings including an off-shell target*

**3 independent variables**

---

+ **3** : nucleus rest frame

+ **1** : off-shell nucleon

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**7 independent variables**

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+ **3** : every on-shell particle

→ we can fix incoming energy ( $E_\nu$ )

→ the cross section is rotationally invariant ( $\phi_\mu$ )

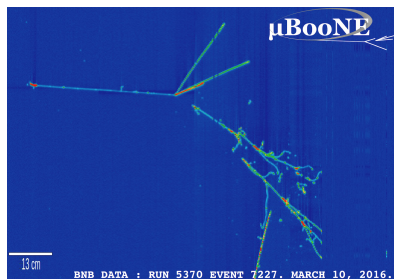
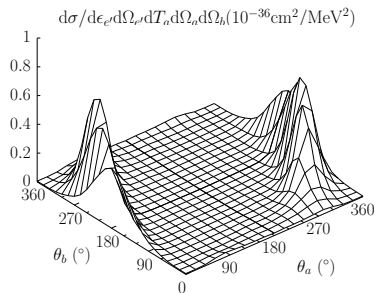
→ the final formula is at least 5-dimensional

# Computing $\nu A$ cross section

Monte Carlo generator

- generate **events**
- cover **whole phase space**
- useful but **approximated**

e.g. **NuWro**



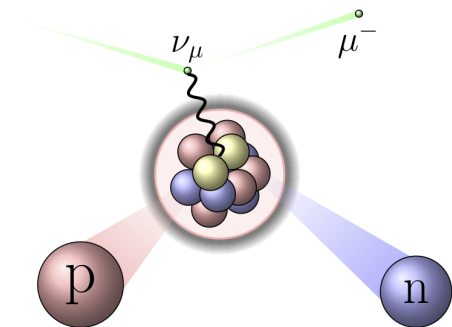
Detailed calculation

- compute **cross sections**
- **fixed kinematics**
- precise but **expensive**

e.g. **Ghent group**

# Contents

- History of 2p2h modeling
- Theoretical formalism of the Ghent group
  - Kinematics
  - Nucleon wave functions
  - Short-range correlations
  - Meson-exchange currents
- Experimental prospects



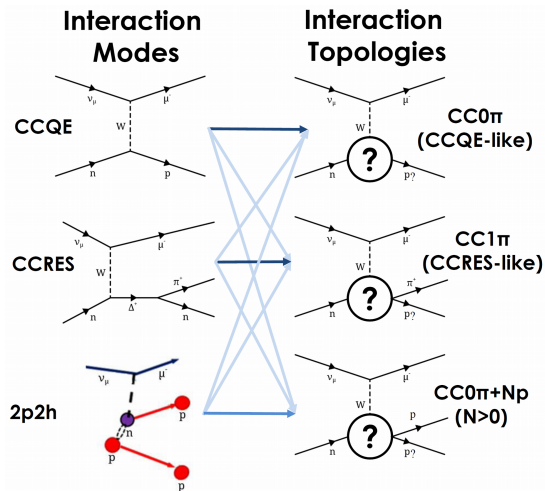
T. Van Cuyck, N. Jachowicz, R. González-Jiménez et al., Phys.Rev.C 95 (2017) 054611

T. Van Cuyck, N. Jachowicz, R. González-Jiménez et al., Phys.Rev.C 94 (2016) 024611



# The MiniBooNE puzzle

An attempt to make a **pure CCQE** measurement...

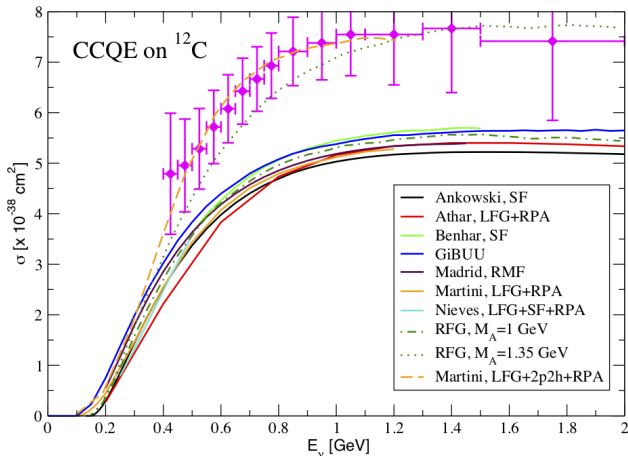


S. Dolan

# The MiniBooNE puzzle

An attempt to make a **pure CCQE** measurement...

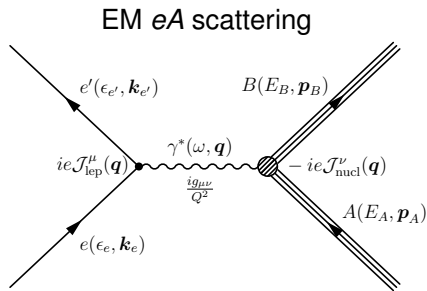
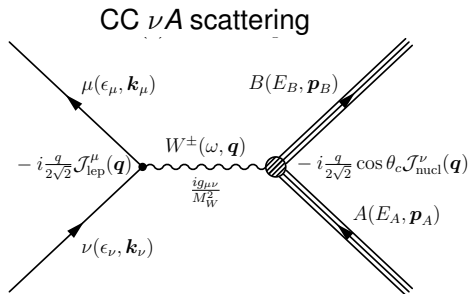
→ suffered from huge **model dependencies**



L. Alvarez-Ruso, Nucl.Phys.B Proc.Suppl. 229-232 (2012) 167-173 (Neutrino 2010)

# The theoretical framework: language of response functions

# Cross section formula



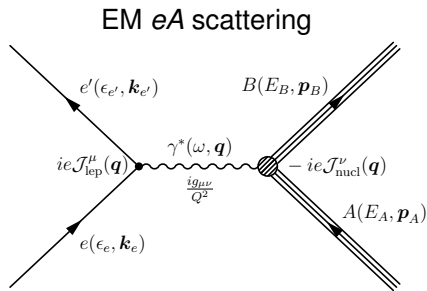
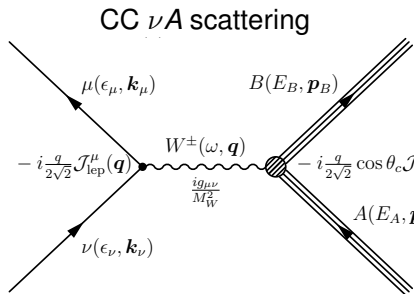
Currents:

$$\mathcal{J}_\mu^{\text{lep}}(q) \equiv \bar{u}(k_f, s_f) \hat{J}_\mu^{\text{lep}} u(k_i, s_i) = \bar{u}(k_f, s_f) \gamma_\mu (1 + h\gamma^5) u(k_i, s_i)$$

$$\mathcal{J}_\mu^{\text{nuc}}(q) \equiv \langle \Psi_f | \hat{J}_\mu^{\text{nuc}} | \Psi_i \rangle$$

where  $h = 0$  for (unpolarized) electrons, and  $h = -(+)$  for (anti)neutrinos

# Cross section formula

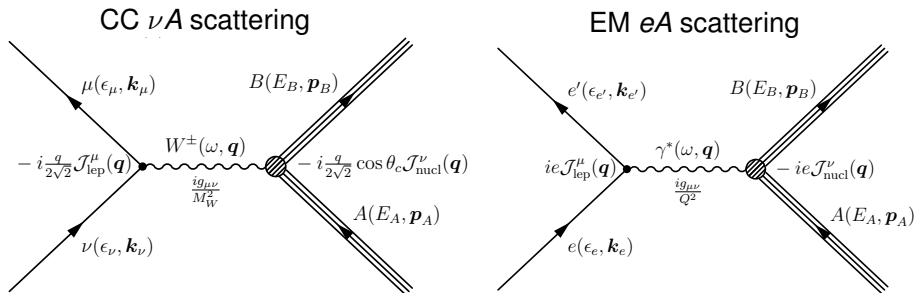


Matrix elements:

$$\mathcal{M}_{fi}^W = -i \frac{G_F}{\sqrt{2}} \cos \theta_c \mathcal{J}_\nu^{\text{lep}}(q) \mathcal{J}_{\text{nuc}}^\nu(q)$$

$$\mathcal{M}_{fi}^\gamma = -i \frac{e^2}{Q^2} \mathcal{J}_\nu^{\text{lep}}(q) \mathcal{J}_{\text{nuc}}^\nu(q)$$

# Cross section formula

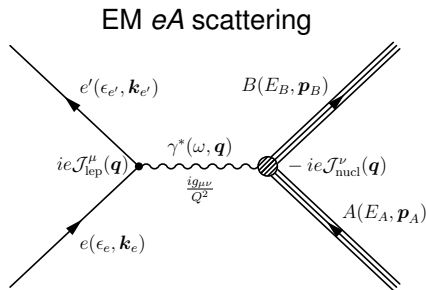
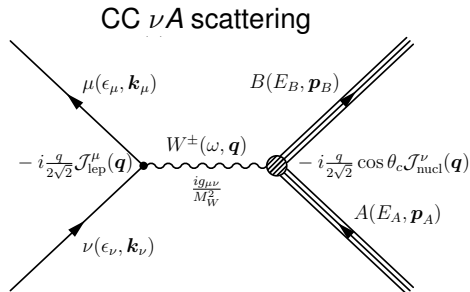


The cross section is proportional to the square:

$$\overline{\sum_{if} |\mathcal{M}_{fi}^W|^2} = \frac{G_F^2}{2} \cos^2 \theta_c L_{\mu\nu} H^{\mu\nu}$$

$$\overline{\sum_{if} |\mathcal{M}_{fi}^\gamma|^2} = \frac{e^4}{4Q^2} L_{\mu\nu} H^{\mu\nu}$$

# Cross section formula

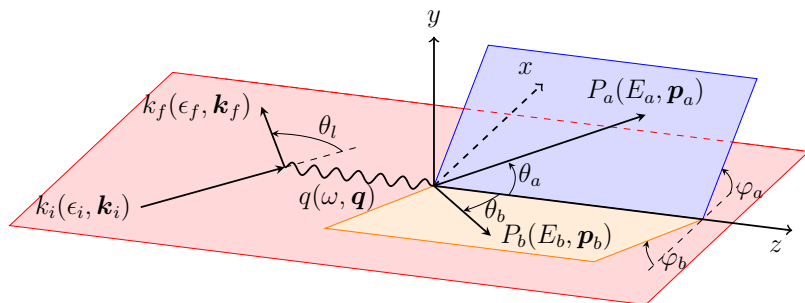


Leptonic tensor:

$$L_{\mu\nu} \propto \left( k_{i,\mu} k_{f,\nu} + k_{f,\nu} k_{i,\mu} + g_{\mu\nu} m_i m_f - g_{\mu\nu} k_i \cdot k_f - ih \epsilon_{\mu\nu\alpha\beta} k_i^\alpha k_f^\beta \right)$$

the axial term  $(-ih \epsilon_{\mu\nu\alpha\beta} k_i^\alpha k_f^\beta)$  drops down for electrons ( $h = 0$ )

# Cross section formula



In such frame of reference:

$$L_{\mu\nu} W^{\mu\nu} = \frac{2\epsilon_i\epsilon_f}{m_i m_f} [v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC} \\ + v_{TL} W_{TL} + h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'})]$$



# Lepton responses

$$v_{CC} = 1 + \zeta \cos \theta$$

$$v_{CL} = -\left(\frac{\omega}{q}(1 + \zeta \cos \theta) + \frac{m_f^2}{\epsilon_f q}\right)$$

$$v_{LL} = 1 + \zeta \cos \theta - \frac{2\epsilon_i \epsilon_f}{q^2} \zeta^2 \sin^2 \theta$$

$$v_T = 1 - \zeta \cos \theta + \frac{\epsilon_i \epsilon_f}{q^2} \zeta^2 \sin^2 \theta$$

$$v_{TT} = -\frac{\epsilon_i \epsilon_f}{q^2} \zeta^2 \sin^2 \theta$$

$$v_{TC} = -\frac{\sin \theta}{\sqrt{2}q} \zeta (\epsilon_i + \epsilon_f)$$

$$v_{TL} = \frac{\sin \theta}{\sqrt{2}q^2} \zeta (\epsilon_i^2 - \epsilon_f^2 + m_f^2)$$

$$v_{T'} = \frac{\epsilon_i + \epsilon_f}{q} (1 - \zeta \cos \theta) - \frac{m_f^2}{\epsilon_f q}$$

$$v_{TC'} = -\frac{\sin \theta}{\sqrt{2}} \zeta$$

$$v_{TL'} = \frac{\omega \sin \theta}{q \sqrt{2}} \zeta$$

→ **dimensionless kinematical factors**

# One-nucleon knockout

$$\frac{d\sigma}{dE_{e'} d\Omega_{e'}} = 4\pi\sigma^X \zeta f_{rec}^{-1} [v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + hv_{T'} W_{T'}],$$

with  $v_i$  and  $\sigma^X$  containing leptonic information, e.g.

$$\sigma^{\text{Mott}} = \left( \frac{\alpha \cos(\theta_{e'}/2)}{2E_e \sin^2(\theta_{e'}/2)} \right)^2, \quad \sigma^W = \left( \frac{G_F \cos\theta_c E_\mu}{2\pi} \right)^2,$$

and the response functions  $W_i$  containing the nuclear information

$$W_{CC} = |\mathcal{J}_0|^2$$

$$W_{CL} = 2\Re(\mathcal{J}_0 \mathcal{J}_3^\dagger)$$

$$W_{LL} = |\mathcal{J}_3|^2$$

$$W_T = |\mathcal{J}_+|^2 + |\mathcal{J}_-|^2$$

$$W_{T'} = |\mathcal{J}_+|^2 - |\mathcal{J}_-|^2$$

$$\mathcal{J}_0 = \langle \Psi_f | \hat{J}_0(q) | \Psi_i \rangle$$

$$\mathcal{J}_+ = \langle \Psi_f | \hat{J}_+(q) | \Psi_i \rangle$$

$$\mathcal{J}_- = \langle \Psi_f | \hat{J}_-(q) | \Psi_i \rangle$$

$$\mathcal{J}_3 = \langle \Psi_f | \hat{J}_3(q) | \Psi_i \rangle$$

## Two-nucleon knockout

$$\frac{d\sigma}{dE_f' d\Omega_f' dT_a d\Omega_a d\Omega_b} = \sigma^X \zeta g_{rec}^{-1} \\ \times \left[ v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC} \right. \\ \left. + v_{TL} W_{TL} + h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'}) \right],$$

$$W_{TT} = 2\Re \left( \mathcal{J}_+ \mathcal{J}_-^\dagger \right)$$

$$W_{TC} = 2\Re \left( \mathcal{J}_0 (\mathcal{J}_+ - \mathcal{J}_-)^\dagger \right)$$

$$W_{TL} = 2\Re \left( \mathcal{J}_3 (\mathcal{J}_+ - \mathcal{J}_-)^\dagger \right)$$

$$W_{TC'} = 2\Re \left( \mathcal{J}_0 (\mathcal{J}_+ + \mathcal{J}_-)^\dagger \right)$$

$$W_{TL'} = 2\Re \left( \mathcal{J}_3 (\mathcal{J}_+ + \mathcal{J}_-)^\dagger \right)$$

$$\mathcal{J}_0 = \langle \Psi_f | \hat{J}_0(q) | \Psi_i \rangle$$

$$\mathcal{J}_+ = \langle \Psi_f | \hat{J}_+(q) | \Psi_i \rangle$$

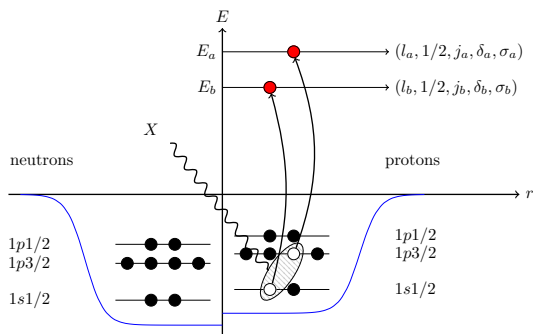
$$\mathcal{J}_- = \langle \Psi_f | \hat{J}_-(q) | \Psi_i \rangle$$

$$\mathcal{J}_3 = \langle \Psi_f | \hat{J}_3(q) | \Psi_i \rangle$$

→ integrate over outgoing nucleons  $\int dT_a d\Omega_a d\Omega_b$

# The theoretical framework: nuclear modeling

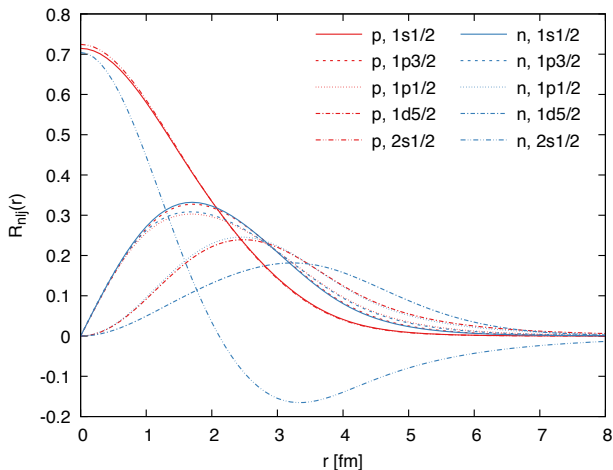
# Nuclear model: initial state



- Ground state nucleus is an **independent-particle model (IPM)**
  - **Mean-field potential** results in a **shell model**
  - Calculated with a **Hartree-Fock (HF) approximation** using a Skyrme NN force (SkE2)
  - Accounts for **binding energies** and **nuclear structure**

# Nuclear model: initial state

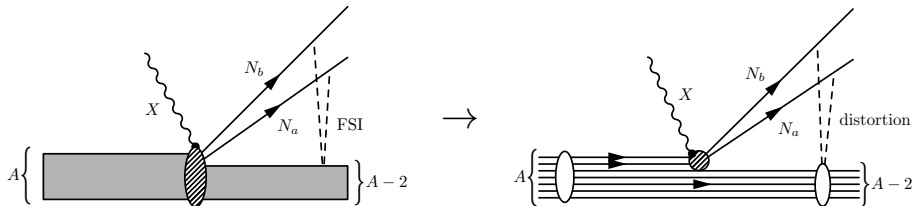
→ we iteratively solve a **radial Schrödinger equation** for  $R_{ljm}$



→ **carbon wave functions** for particular shells

# Nuclear model: final state

- **Continuum wave functions** are calculated using the **same NN potential**
  - **Orthogonality is preserved** between initial and final states
  - **Distortion effects** of the residual nucleus on the ejected nucleons are incorporated
  - Pauli-blocking effects included inherently



# Multipole expansion

→ we perform **non-relativistic reduction of operators**

→ simplify integrals with **multipole expansion**

$$\hat{\rho}(\mathbf{q}) \rightarrow \hat{M}_{JM}^{\text{Coul}}(q) = \int d\mathbf{r} [j_J(qr) Y_{JM}(\Omega_r)] \hat{\rho}(\mathbf{r})$$

$$\hat{J}_3(\mathbf{q}) \rightarrow \hat{L}_{JM}^{\text{long}}(q) = \frac{i}{q} \int d\mathbf{r} [\nabla(j_J(qr) Y_{JM}(\Omega_r))] \cdot \hat{J}(\mathbf{r})$$

$$\hat{J}_{\pm}(\mathbf{q}) \rightarrow \hat{T}_{JM}^{\text{elec}}(q) = \frac{1}{q} \int d\mathbf{r} \left[ \nabla \times (j_J(qr) \mathbf{Y}_{J(J,q)}^M(\Omega_r)) \right] \cdot \hat{J}(\mathbf{r})$$

$$\rightarrow \hat{T}_{JM}^{\text{magn}}(q) = \int d\mathbf{r} [j_J(qr) \mathbf{Y}_{J(J,q)}^M(\Omega_r)] \cdot \hat{J}(\mathbf{r})$$

→ summation over  $J$  increases the accuracy of our results



# Nuclear currents in the IA

$$\hat{\rho}_V(\mathbf{r}) = \sum_i^A F_1(Q^2) \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \tau_{\pm}(i)$$

$$\hat{\rho}_A(\mathbf{r}) = \sum_i^A \frac{G_A(Q^2)}{2m_N i} \sigma_i \cdot \left[ \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \vec{\nabla}_i - \overleftarrow{\nabla}_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \right] \tau_{\pm}(i)$$

$$\hat{J}_V(\mathbf{r}) = \hat{J}_{\text{con}}(\mathbf{r}) + \hat{J}_{\text{mag}}(\mathbf{r})$$

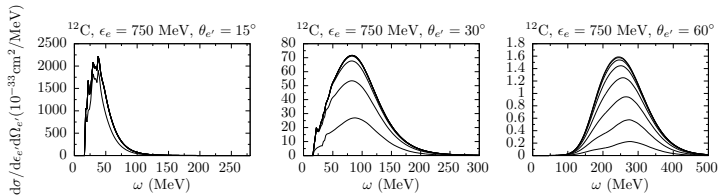
$$= \sum_i^A \frac{F_1(Q^2)}{2m_N i} \left[ \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \vec{\nabla}_i - \overleftarrow{\nabla}_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \right] \tau_{\pm}(i)$$

$$+ \sum_i^A \frac{F_1(Q^2) + F_2(Q^2)}{2m_N} \left( \vec{\nabla} \times \sigma_i \right) \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \tau_{\pm}(i)$$

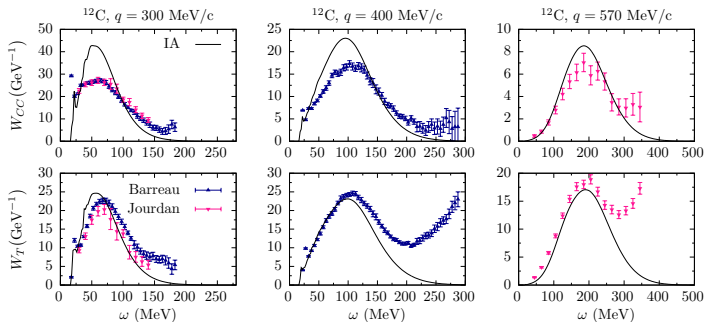
$$\hat{J}_A(\mathbf{r}) = \sum_i^A G_A(Q^2) \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \sigma_i \tau_{\pm}(i)$$

# One-nucleon knockout

→ multipoles contribution

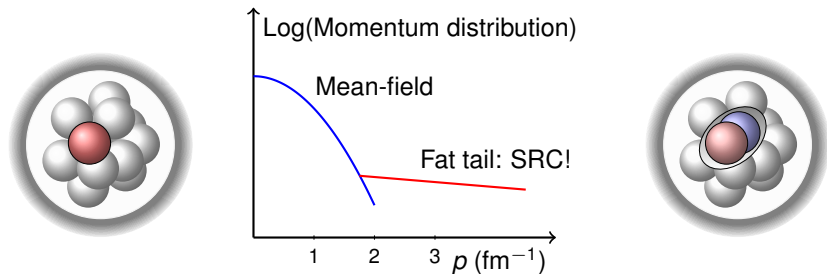


→ comparison to electron scattering data



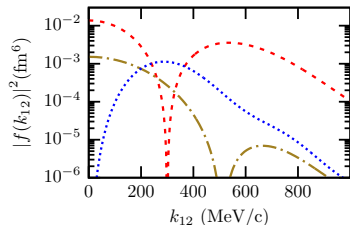
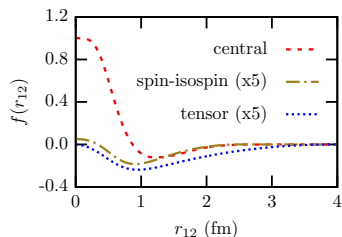
# Short-range correlations

**Fat tails in the single-nucleon momentum distribution** cannot be explained within an independent-particle model (IPM)



- **Nucleons occur in pairs** with **high relative momenta** and **low center-of-mass momenta** (SRC pairs)
- **Mean-field**: momenta **below**  $k_F$ , **SRC pairs**: momenta **above**  $k_F$
- A signature of SRC is **back-to-back**  $2N$  knockout
- SRC also have an effect on  $1N$  knockout

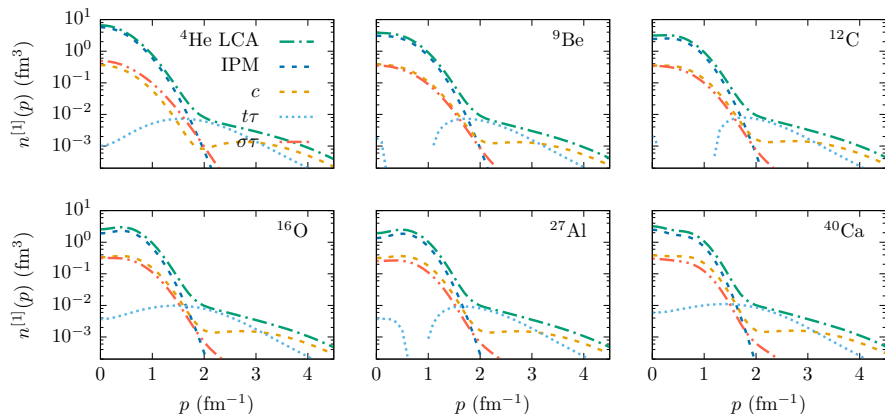
# Short-range correlations



- The correlations have a **short range**:  $f(r_{ij}) \rightarrow 0$  at  $r_{ij} > 3$  fm
- Tensor correlation function dominates for intermediate relative momenta 200 – 400 MeV/c
- Central correlation function dominates at high relative momenta
- Spin-isospin correlation function overall relatively small
- These correlation functions are input

(Gearhart, 1994), (Pieper, Wiringa, and Pandharipande, 1992)

# Short-range correlations



Single-nucleon momentum distribution

# Short-range correlations

Correlated wave functions  $|\Psi\rangle$  are constructed by acting with a many-body correlation operator  $\hat{\mathcal{G}}$  on the uncorrelated Hartree-Fock wave functions  $|\Phi\rangle$

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}} \hat{\mathcal{G}} |\Phi\rangle, \quad \text{with} \quad \mathcal{N} = \langle \Phi | \hat{\mathcal{G}}^\dagger \hat{\mathcal{G}} | \Phi \rangle$$

The central ( $c$ ), tensor ( $t_\tau$ ) and spin-isospin ( $\sigma\tau$ ) correlations are responsible for majority of the strength

$$\hat{\mathcal{G}} \approx \hat{\mathcal{S}} \left( \prod_{i < j}^A [1 + \hat{l}(i, j)] \right)$$

with  $\hat{\mathcal{S}}$  the symmetrization operator and

$$\hat{l}(i, j) = -g_c(r_{ij}) + f_{t_\tau}(r_{ij}) \hat{\mathcal{S}}_{ij} (\vec{\tau}_i \cdot \vec{\tau}_j) + f_{\sigma\tau}(r_{ij}) (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{\tau}_i \cdot \vec{\tau}_j).$$

$g_c(r_{ij})$ ,  $f_{t_\tau}(r_{ij})$  and  $f_{\sigma\tau}(r_{ij})$  are the respective correlation functions

Correlation functions: (Gearhart, 1994), (Pieper, Wiringa, and Pandharipande, 1992)

# Short-range correlations

Transition matrix elements between **correlated states**  $|\Psi\rangle$  can be written as ones between **uncorrelated states**  $|\Phi\rangle$ , with an effective transition operator

$$\langle \Psi_f | \hat{J}_\mu^{\text{nucl}} | \Psi_i \rangle = \frac{1}{\sqrt{\mathcal{N}_i \mathcal{N}_f}} \langle \Phi_f | \hat{J}_\mu^{\text{eff}} | \Phi_i \rangle,$$

with

$$\hat{J}_\mu^{\text{eff}} = \hat{\mathcal{G}}^\dagger \hat{J}_\mu^{\text{nucl}} \hat{\mathcal{G}} = \left( \prod_{j < k}^A [1 + \hat{l}(j, k)] \right)^\dagger \hat{J}_\mu^{\text{nucl}} \left( \prod_{l < m}^A [1 + \hat{l}(l, m)] \right).$$

In the IA, the many-body nuclear current can be written as a sum of one-body operators

$$\hat{J}_\lambda^{\text{eff}} = \left( \prod_{j < k}^A [1 + \hat{l}(j, k)] \right)^\dagger \sum_{i=1}^A \hat{J}_\lambda^{[1]}(i) \left( \prod_{l < m}^A [1 + \hat{l}(l, m)] \right).$$

# Short-range correlations

Use the fact that SRC is a **short-range** phenomenon

- Terms linear in the correlation operator are retained
- $A$ -body operator → 2-body operator

$$\hat{J}_\lambda^{\text{eff}} \approx \underbrace{\sum_{i=1}^A \hat{J}_\lambda^{[1]}(i)}_{\text{one-body (IA)}} + \underbrace{\sum_{i < j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j)}_{\text{two-body (SRC)}} + \left[ \sum_{i < j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) \right]^\dagger$$

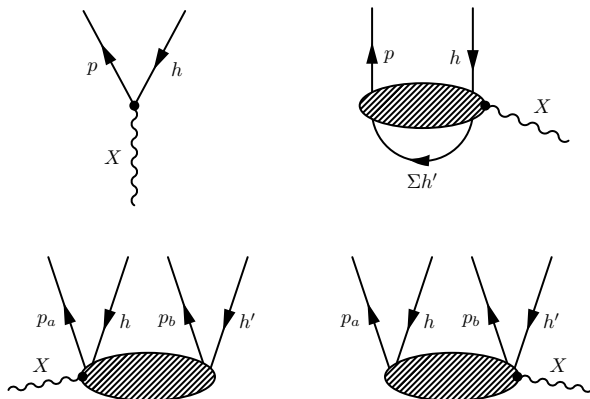
where

$$\hat{J}_\lambda^{[1],\text{in}}(i,j) = \left[ \hat{J}_\lambda^{[1]}(i) + \hat{J}_\lambda^{[1]}(j) \right] \hat{t}(i,j)$$

- **Effective nuclear current is the sum of a one-body (IA) and two-body (SRC) current**

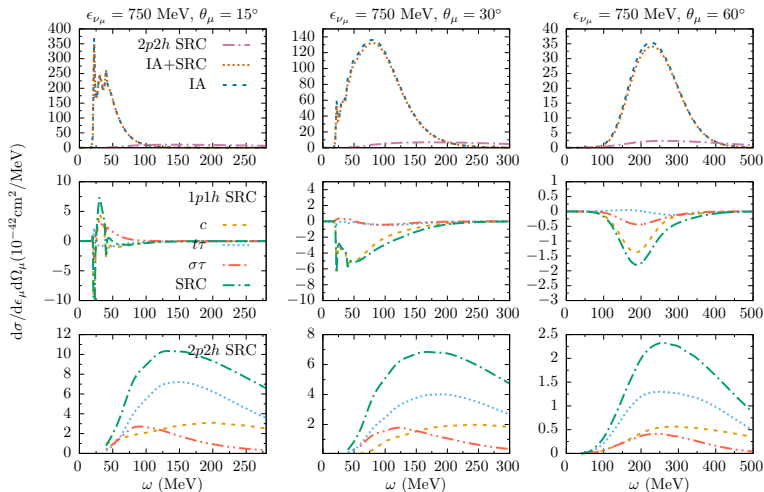


# Short-range correlations



The 1p1h (top) and 2p2h (bottom) diagrams considered. The top left diagram shows the 1p1h channel in the IA.

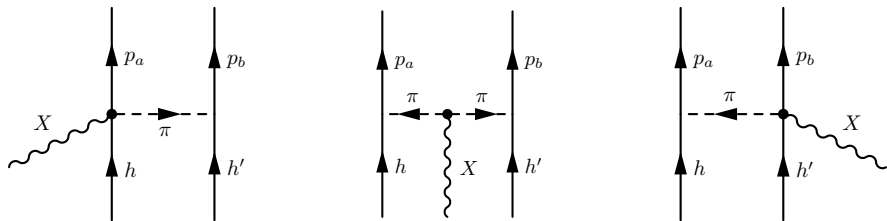
# SRC results - Inclusive $^{12}\text{C}(\nu_\mu, \mu^-)$



→ Small decrease of  $1p1h$  channel due to SRCs

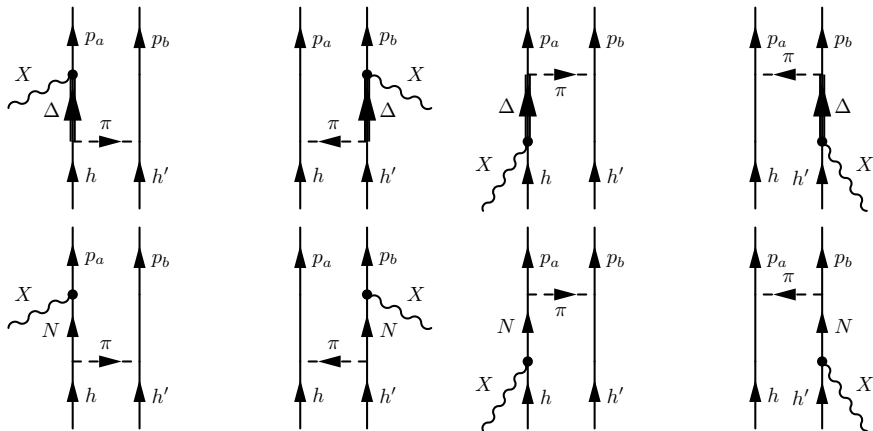
→ Inclusive  $2p2h$  appears as a broad background to  $1p1h$

# Meson-exchange currents



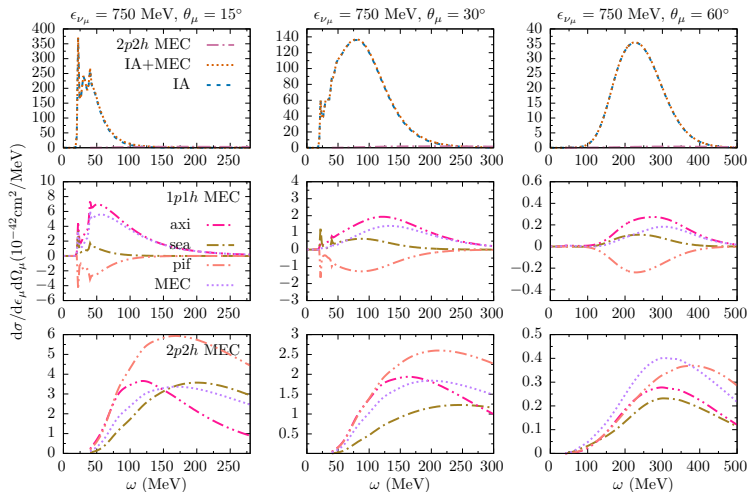
The seagull and pion-in-flight currents.

# Meson-exchange currents



The  $\Delta$  currents (top) and correlation currents (bottom).

# MEC results - Inclusive $^{12}\text{C}(\nu_\mu, \mu^-)$



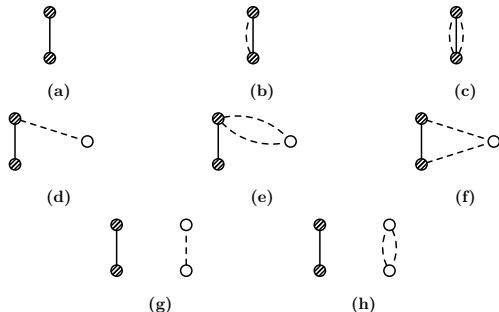
→ Small increase of  $1p1h$  channel due to MECs

→ Inclusive  $2p2h$  appears as a broad background to  $1p1h$

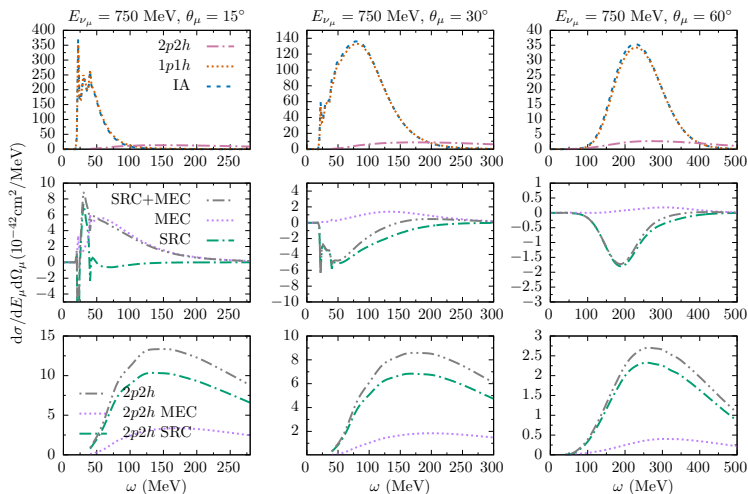
# SRS + MEC

Extend the current model with MECs

$$\hat{J}_\lambda^{\text{eff}} = \underbrace{\sum_{i=1}^A \hat{J}_\lambda^{[1]}(i)}_{\text{one-body(IA)}} + \underbrace{\sum_{i<j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) + \left[ \sum_{i<j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) \right]^\dagger}_{\text{two-body(SRC)}} + \underbrace{\sum_{i<j}^a \hat{J}_\lambda^{[2],\text{sea}}(i,j) + \sum_{i<j}^a \hat{J}_\lambda^{[2],\text{pif}}(i,j)}_{\text{two-body(MEC)}}$$

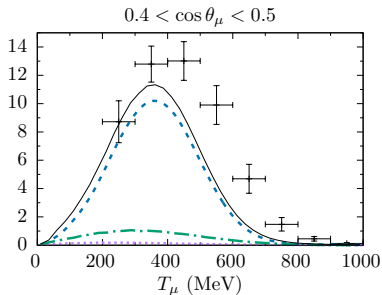
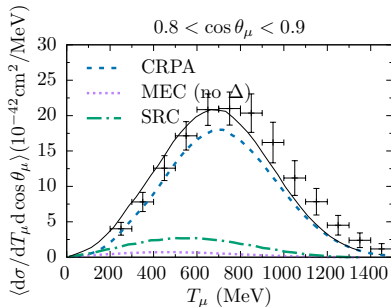
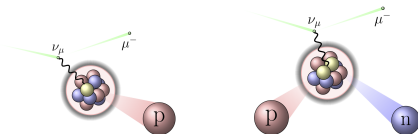


# SRC + MEC results - Inclusive $^{12}\text{C}(\nu_\mu, \mu^-)$



- Effect of MECs largest for small  $\theta_\mu$ , SRCs for larger  $\theta_\mu$  in  $1p1h$  channel
- Inclusive  $2p2h$  appears as a broad background to  $1p1h$

# Comparison with MiniBooNE data

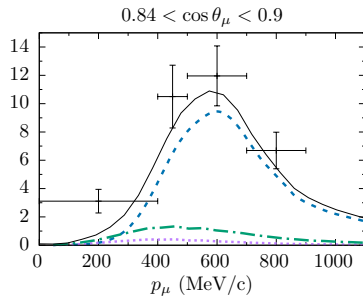
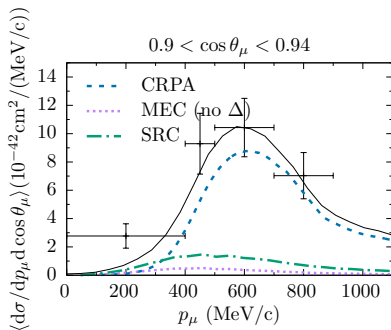
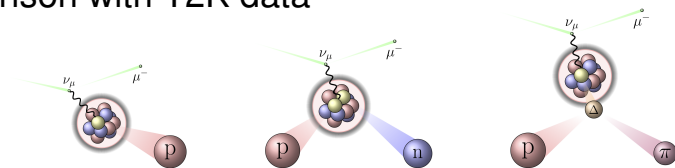


MiniBooNE 'CCQE-like' data from Phys.Rev.D 81 (2010) 092005

CRPA results are from Phys.Rev.C 94 (2016) 054609



# Comparison with T2K data

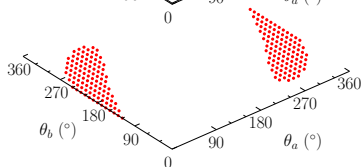
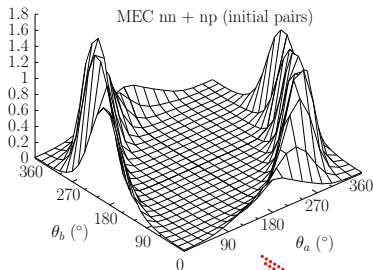
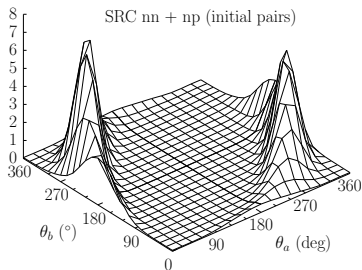


Inclusive T2K data from Phys.Rev.D 87 (2013) 092003

CRPA results are from Phys.Rev.C 94 (2016) 054609

# Exclusive $A(\nu_\mu, \mu^- N_a N_b)$

$$d\sigma/d\epsilon_\mu d\Omega_\mu dT_a d\Omega_a d\Omega_b (10^{-45} \text{cm}^2/\text{MeV}^2)$$



The  $^{12}\text{C}(\nu_\mu, \mu^- N_a N_b)$  cross section at  $\epsilon_{\nu_\mu} = 750$  MeV,  $\epsilon_\mu = 550$  MeV,  $\theta_\mu = 15^\circ$  and  $T_p = 50$  MeV for in-plane kinematics ( $q = 268$  MeV/c,  $x_B = 0.08$ ). The bottom panel shows  $P_{12} < 300$  MeV/c.

# Summary

- The **Ghent group** provides a **powerful model** capable of calculating various contributions to the **2p2h final states**
- The **MEC calculation misses  $\Delta$ -currents** and needs to be further developed
- Efforts are done to **implement** such model in **Monte Carlo event generators** so it can be used in **experimental analyses**

# Collaborators

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## Ghent group

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- Alexis Nikolakopoulos
- Jannes Nys
- Vishvas Pandey
- Tom Van Cuyck
- Nils Van Dessel

*and many more...*