Electroweak single-pion production off the nucleon: effective implementation in Monte Carlo event generators

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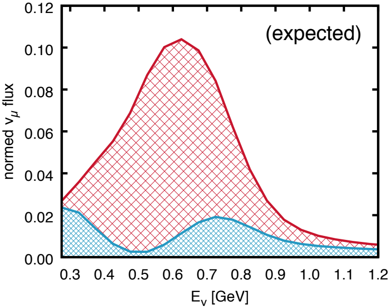
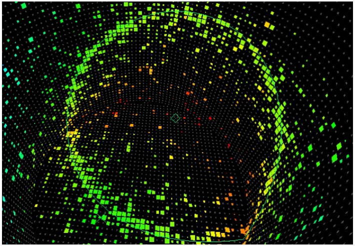
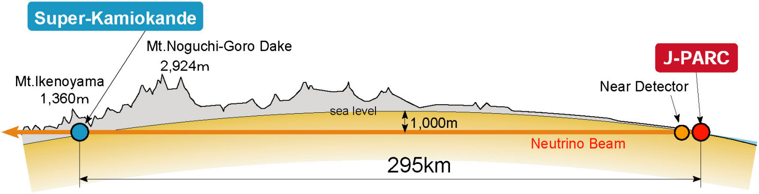
Neutrino oscillation experiments

$$P_{2\mathrm{f}}(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

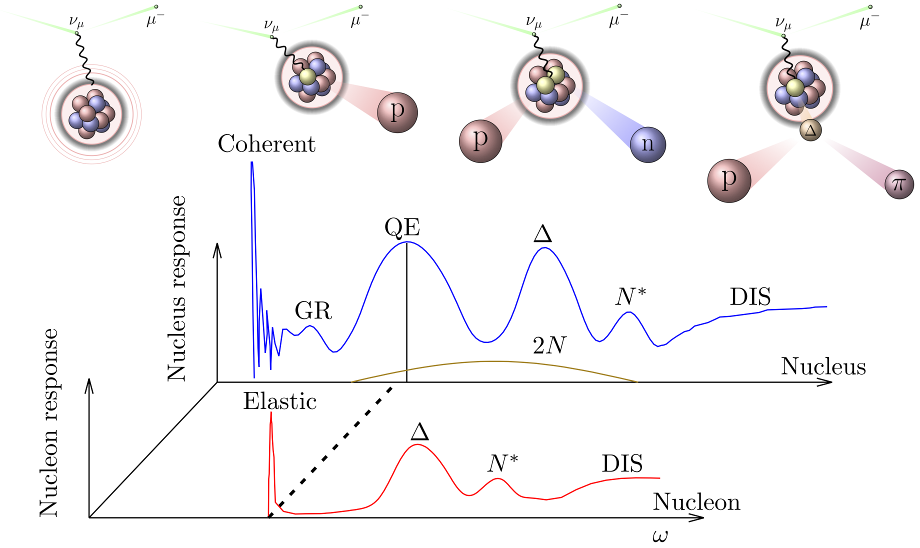




$$E_\nu^{\mathrm{rec}} = \frac{2(M_n - E_B)E_\mu - (E_B^2 - 2M_nE_B + m_\mu^2)}{2[M_n - E_B - E_\mu + |\vec{k}_\mu|\cos\theta_\mu]}$$

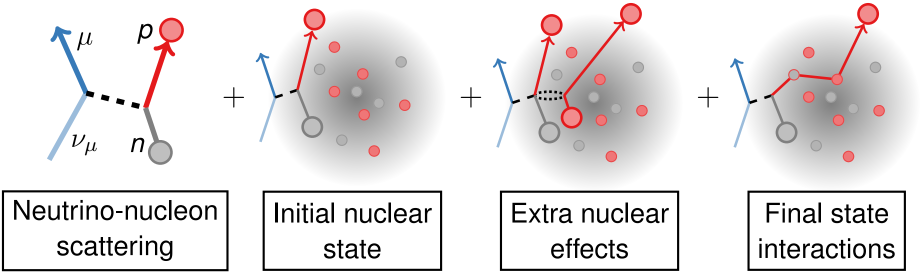


Nuclear response



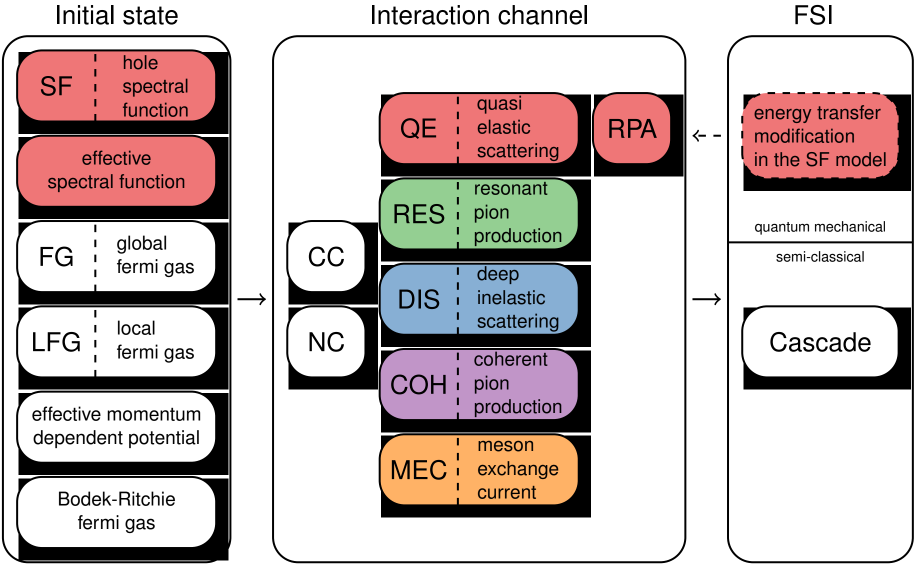
T. Van Cuyck

Cross section in the factorized scheme

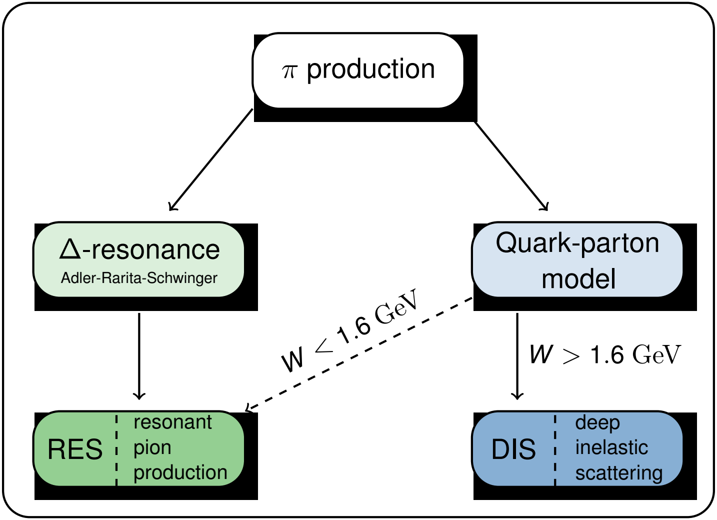


- **Neutrino-nucleon scattering**: elementary interaction cross section
- **Initial nuclear state**: modeling nucleons in the nuclear medium before the weak interaction
- **Extra nuclear effects**: multiple-nucleon interactions or correlations
- **Final state interactions**: in-medium outgoing particle propagation

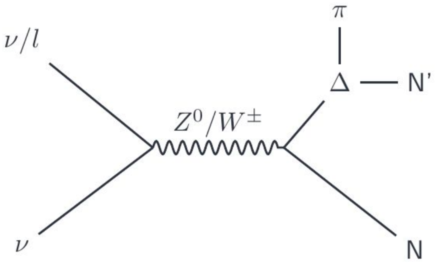
NuWro blueprint



Pion production in NuWro



Resonant pion production



The following channels are considered:

$$\nu + p \to l^- + (\Delta^{++} \to p + \pi^+) \qquad \bar{\nu} + n \to l^+ + (\Delta^- \to n + \pi^-)$$

$$\nu + n \to l^- + (\Delta^{+} \to p + \pi^0 \text{ or } n + \pi^+) \qquad \bar{\nu} + p \to l^+ + (\Delta^0 \to p + \pi^- \text{ or } n + \pi^0)$$

$$\nu(\bar{\nu}) + p \to \nu(\bar{\nu}) + (\Delta^+ \to p + \pi^0 \text{ or } n + \pi^+)$$

$$\nu(\bar{\nu}) + n \to \nu(\bar{\nu}) + (\Delta^0 \to p + \pi^- \text{ or } n + \pi^0)$$

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off a nucleon	Pion production on a nucleus
$\frac{d^2\sigma}{dQ^2 dW}$	$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*}$	$\frac{d^8\sigma}{dQ^2 dW d\Omega_\pi^* dE_m d\vec{p}_m}$

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Adler-Rarita-Schwinger formalism

Double-differential cross section for the Δ **production**:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}W\mathrm{d}Q^2} = G^2 \cos^2\theta_C \frac{W g(W)}{\pi^2 M E_\nu^2} \left[-(Q^2+m^2) V_1 + \frac{V_2}{M^2}\left(2(pq)(pk') \frac{M^2}{2}(Q^2+m^2) \right) \right.$$
$$\left. - \frac{V_3}{M^2}\left(Q^2(kp) - \frac{1}{2}(Q^2+m^2)(pq) \right) + \frac{V_4}{m^2}\frac{m^2}{2} - 2\frac{V_5}{M^2} m^2 (kp) \right]$$

where V_i are structure functions made of **hadronic tensor elements** and

$$g(W) = \frac{\Gamma_\Delta/2}{(W-M_\Delta)^2 + \Gamma_\Delta^2/4}$$

is the **Breit-Wigner** formula introducing the Δ **width** (Γ_Δ)

S. L. Adler, Annals Phys. 50 (1968) 189-311; S. L. Adler, Phys.Rev. D12 (1975) 2644

Rarita-Schwinger field Ψ_μ

→ The **final hadronic state** is a $\frac{3}{2}$-**spin resonance** described as a **Rarita-Schwinger field**

→ The **transition** from the **nucleon** to, e.g., Δ^{++} **state** is given as a matrix element of the **weak hadronic current**: $\mathcal{J}_\mu^{CC} = \mathcal{J}_\mu^V + \mathcal{J}_\mu^A$

$$\left\langle \Delta^{++}(p') \right| \mathcal{J}_\mu^V \left| N(p) \right\rangle =$$
$$\sqrt{3}\bar{\Psi}_\lambda(p') \left[g_\mu^\lambda \left(\frac{C_3^V(Q^2)}{M}\gamma_\nu + \frac{C_4^V(Q^2)}{M^2}p'_\nu + \frac{C_5^V(Q^2)}{M^2}p_\nu \right) q^\nu - q^\lambda \left(\frac{C_3^V(Q^2)}{M}\gamma_\nu + \frac{C_4^V}{M^2}p'_\nu + \frac{C_5^V(Q^2)}{M^2} \right) \right] \gamma_5 u(p)$$

$$\left\langle \Delta^{++}(p') \right| \mathcal{J}_\mu^A \left| N(p) \right\rangle =$$
$$\sqrt{3}\bar{\Psi}_\lambda(p') \left[g_\mu^\lambda \left(\gamma_\nu \frac{C_3^A(Q^2)}{M} + \frac{C_4^A(Q^2)}{M^2} \right) q^\nu - q^\lambda \left(\frac{C_3^A(Q^2)}{M}\gamma_\mu + \frac{C_4^A(Q^2)}{M^2}p'_\mu \right) + g_\mu^\lambda C_5^A(Q^2) + \frac{q^\lambda q_\mu}{M^2} C_6^A(Q^2) \right] u(p)$$

Hadronic tensor $W_{\mu\nu}$

Defined as

$$W_{\mu\nu} = \frac{1}{4MM_\Delta}\frac{1}{2}\sum_{\text{spin}} \langle \Delta^{++}(p') | \mathcal{J}_\mu^{CC} | N(p) \rangle \langle \Delta^{++}(p') | \mathcal{J}_\nu^{CC} | N(p) \rangle^* \times \frac{\Gamma_\Delta/2}{(W - M_\Delta)^2 + \Gamma_\Delta^2/4}$$

$\Gamma_\Delta(W)$ is the Δ width, for which we assume the P-wave (l=1) expression

$$\Gamma_\Delta = \Gamma_0 \left(\frac{q_{cm}(W)}{q_{cm}(W_\Delta)} \right)^{2l+1} \frac{M_\Delta}{W}$$

with

$$q_{cm}(W) = \sqrt{\left(\frac{W^2 + M^2 - m_\pi^2}{2W} \right)^2 - M^2}$$

$\Gamma_0 = 120\ \mathrm{MeV}$, $M_\Delta = 1232\ \mathrm{MeV}$, $m_\pi = 139.57\ \mathrm{MeV}$

Form Factors

Elementary information lies in **vector** and **axial** **form factors** ${C^{V,A}}_i$

There are **several parametrizations available** in NuWro

Our default choice:

C_5^A axial form factor from bubble chamber experiments

[K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys.Rev. D80 (2009) 093001]

$\rightarrow$ A **consistent fit** to **both ANL** and **BNL** data

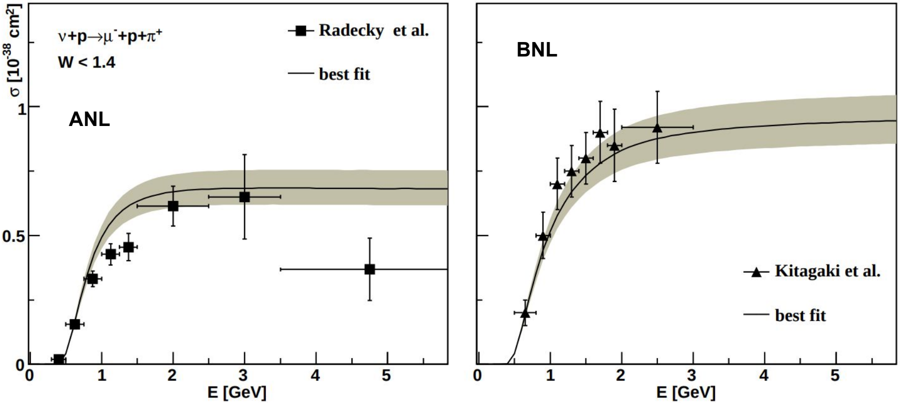
$\rightarrow$ **Only** Δ^{++} channel **assuming** there is **no background**

$\rightarrow$ **Consistency with NuWro**: only Δ^{++} in the given channel

Dipole parametrization, $M_A = 0.94\ \mathrm{GeV}$, $C_5^A(0) = 1.19$!

+ vector part from [O. Lalakulich, E. A. Paschos, G. Piranishvili, Phys.Rev. D 74 (2006) 014009]

Comparison with ANL/BNL data



→ **a simultaneus fit to ANL and BNL that shows their consistency !**

K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys.Rev. D80 (2009) 093001

Dimensionality of the problem

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$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W}$	$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi}$	$\frac{\mathrm{d}^8\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi\mathrm{d}E_m\mathrm{d}\vec{p}_m}$

+2

Include angular information about the Δ decay (Ω^*_π)

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Pion production off a nucleon

To produce an event, one needs **information about** the **produced pion**

Delta decays in the **hadronic CMS**:

$$\frac{\mathrm{d}^2\sigma_\Delta}{\mathrm{d}Q^2\mathrm{d}W} \to \frac{\mathrm{d}^4\sigma_\pi}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*} \times f_\Delta(\Omega_\pi^*)$$

Pion angular distributions are essential to **generate** the **kinematics**

In **NuWro**, it is taken from **experimental results** (ANL or BNL):

S.J. Barish et al., Phys.Rev. D19 (1979) 2511

G.M. Radecky et al., Phys.Rev. D25 (1982) 1161

T. Kitagaki et al., Phys.Rev. D34 (1986) 2554

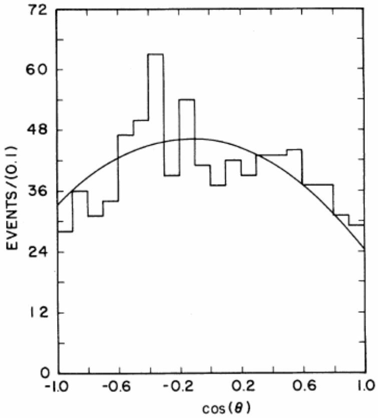
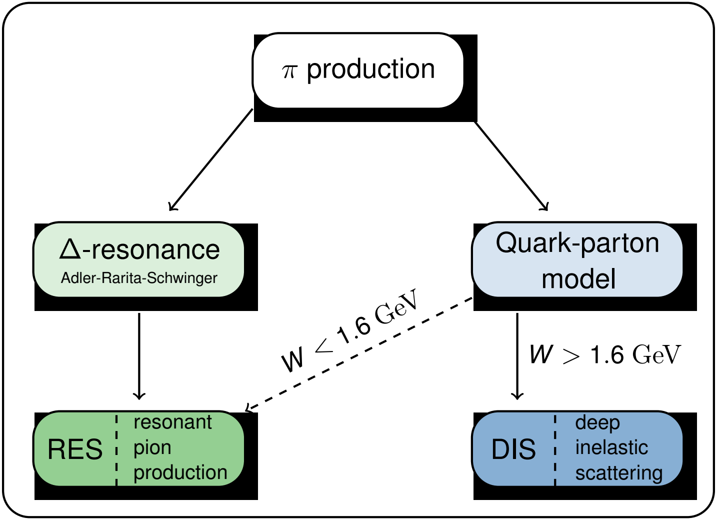


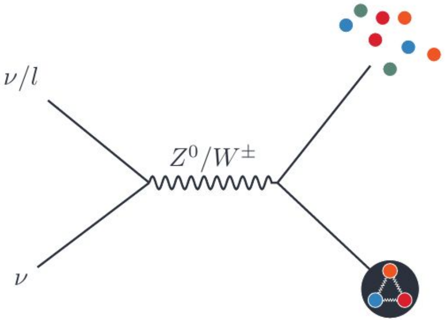
FIG. 15. Distribution of events in the pion polar angle cosθ for the final state $\mu^- p \pi^+$, with $M(p\pi^+) < 1.4$ GeV. The curve is the area-normalized prediction of the Adler model.

Radecky et al. [ANL Collaboration], PRD 25 (1982) 1161

Pion production in NuWro



Deep inelastic scattering in NuWro



Events with invariant mass $W > 1.6\ \mathrm{GeV}$ are considered within the quark-parton model and labeled as DIS:

$$\nu + N \rightarrow l^- + X \qquad \bar{\nu} + N \rightarrow l^+ + X$$

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + X$$

DIS cross section

Double-differential cross section expressed in terms of $x = Q^2/2M\omega$, $y = \omega/E_\nu$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{G^2 M E_\nu}{\pi(1+Q^2/M_{W,Z}^2)^2}\left[y\left(xy + \frac{m^2}{2E_\nu M}\right) F_1(x,Q^2) + \left(1 - y - \frac{Mxy}{2E_\nu} - \frac{m^2}{4E_\nu^2} - \frac{m^2}{2ME_\nu x}\right) F_2(x,Q^2) \pm \left(xy\left(1-\frac{y}{2}\right) - y\frac{m^2}{4ME_\nu}\right) F_3(x,Q^2)\right]$$

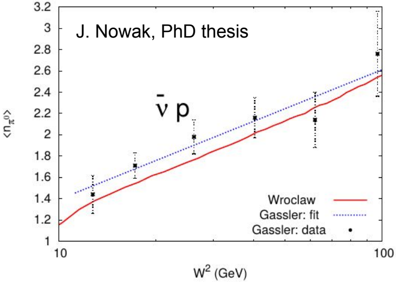
where $F_{1,2,3}$ are expressed by the **parton distribution functions**

$\rightarrow$ **GRV95 parametrization + low-Q^2 Bodek-Yang corrections**

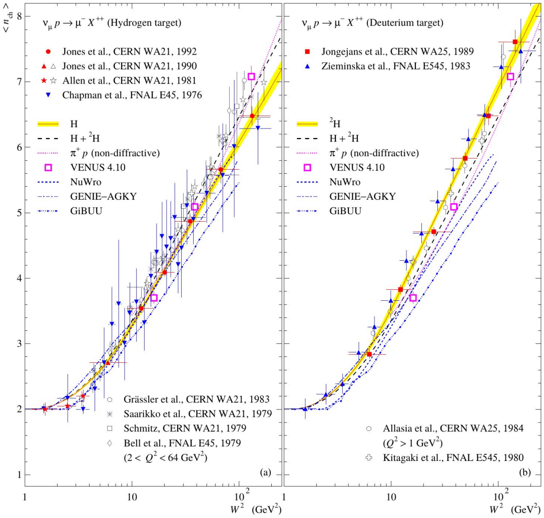
Hadronization

→ Hard-crafted **parameters tuned** to **experimental data**

→ Performed using **Pythia6** routines



Multiplicity of π^0



Multiplicity of π^+

Transition region & Non-resonant background

The **background extrapolated** from the **DIS** region (SPP + more)

Smooth SPP transition from **RES** to **DIS** in the W range $(1.3, 1.6)\ \mathrm{GeV}$:

$$\frac{\mathrm{d}\sigma^{\mathrm{SPP}}}{\mathrm{d}W} = \frac{\mathrm{d}\sigma^{\Delta}}{\mathrm{d}W}(1 - \alpha(W)) + \frac{\mathrm{d}\sigma^{\mathrm{DIS}}}{\mathrm{d}W} F^{\mathrm{SPP}} \alpha(W)$$

where $\alpha(W)$ assures a **smooth transition** and F^{SPP} is the **fraction** of **single pion production** in DIS

$$\begin{aligned}\alpha(W) = {} & \Theta(W_{min} - W)\frac{W - W_{th}}{W_{min} - W_{th}}\alpha_0 \\ & + \Theta(W_{max} - W)\Theta(W - W_{min})\frac{W - W_{min} + \alpha_0(W_{max} - W)}{W_{max} - W_{min}} \\ & + \Theta(W - W_{max})\end{aligned}$$

channel	$\nu_l\, p \to l^-\, p\, \pi^+$	$\nu_l\, n \to l^-\, n\, \pi^+$	$\nu_l\, n \to l^-\, p\, \pi^0$	$\bar{\nu}_l\, n \to l^+\, n\, \pi^-$	$\bar{\nu}_l\, p \to l^+\, p\, \pi^-$	$\bar{\nu}_l\, p \to l^+\, n\, \pi^0$
α_0	0.0	0.2	0.3	0.0	0.2	0.3

For all NC SPP channels: $\alpha_0 = 0$

Pion production in NuWro

We have

$\rightarrow$ a very good description on the Δ peak

but

$\rightarrow$ an incoherent sum of the resonant part and the background

$\rightarrow$ disentangled pion angular distributions

$\rightarrow$ only one resonance

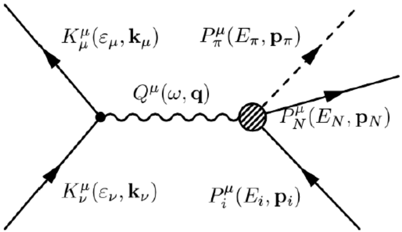
The Hybrid model of the Ghent group

The Hybrid model of the Ghent group

References:

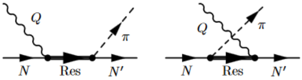
- *Neutrino-induced pion production from nuclei at medium energies*, C. Praet, O. Lalakulich, N. Jachowicz, J. Ryckebusch, Phys. Rev. C79 (2009) 044603, arXiv:0804.2750
- *Electroweak single-pion production off the nucleon: from threshold to high invariant masses*, R. González Jiménez, N. Jachowicz, K. Niewczas, J. Nys, V. Pandey, T. Van Cuyck, N. Van Dessel, Phys. Rev. D95 (2017) 113007, arXiv:1612.05511
- *Pion production within the hybrid-RPWIA model at MiniBooNE and MINERvA kinematics*, R. González Jiménez, K. Niewczas, N. Jachowicz, Phys. Rev. D97 (2018) 093008, arXiv:1710.08374
- *Modeling neutrino-induced charged pion production on water at T2K kinematics*, A. Nikolakopoulos, R. González Jiménez, K. Niewczas, J. Sobczyk, N. Jachowicz, Phys. Rev. D97 (2018) 093008, arXiv:1803.03163
- *Nuclear effects in electron- and neutrino-nucleus scattering within a relativistic quantum mechanical framework*, R. González Jiménez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias, arXiv1904:10696, accepted for publication in PRC

Single pion production on the nucleon (Ghent)



Cfr. PRC 76, 033005 (2007), PRD87, 113009 (2013)

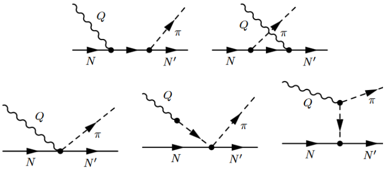
Resonances



P33 (1232), P11(1440), D13 (1520), S11 (1535

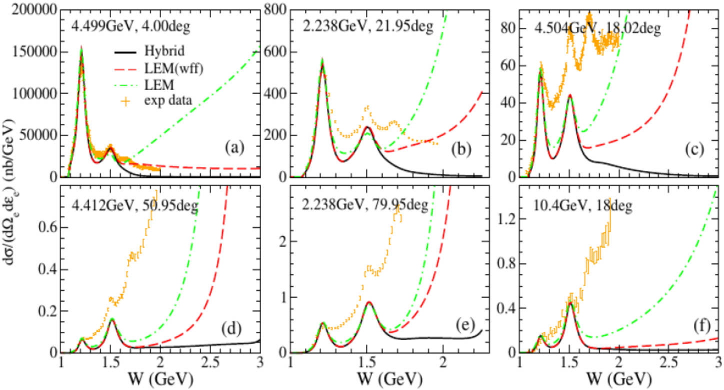
+

ChPT background



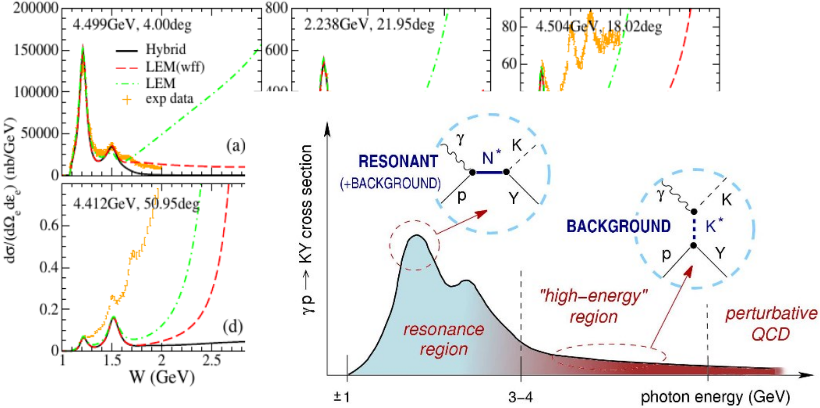
N. Jachowicz

Single pion production on the nucleon (Ghent)



N. Jachowicz

Single pion production on the nucleon (Ghent)

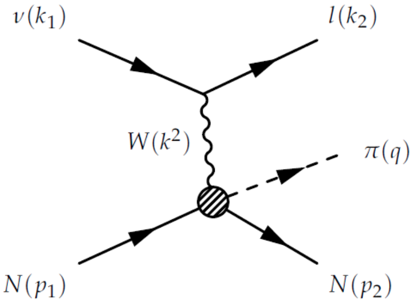


T. Corthals, PhD, UGent 2005

N. Jachowicz

Implementation in Monte Carlo event generators

Single pion production on the nucleon



5 Four vectors = 5x4 = 20 variables

- 4 : on mass shell relations
- 4 : initial nucleon known (at rest)
- 4 : Energy-momentum conservation
- 3 : Freedom to choose reference frame
  And invariance along q
  (known direction of one four vector)

= 5 independent variables

E_ν , $\cos\theta_l$, E_l , Ω_π* or $E_{\nu,}$ Q^2,W, Ω_π*

A. Nikolakopoulos

Single pion production on the nucleon

One can exploit certain properties of such system

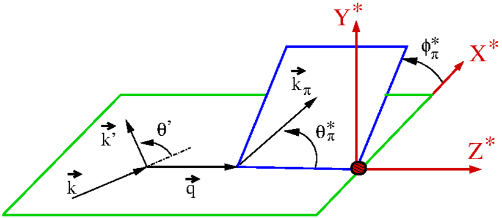
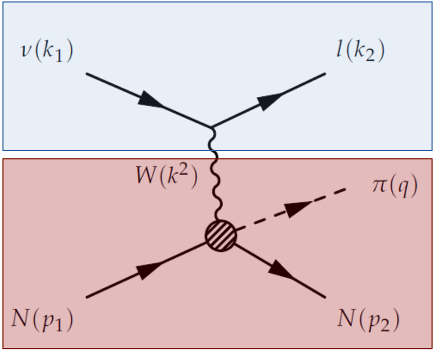


FIG. 3. Definition of the scattering and reaction planes. The $X^*Y^*Z^*$ coordinate axes move along with the CM system of the final pion-nucleon and their orientation has been chosen in such a way that the lepton momenta lie in the $O^*X^*Z^*$ plane with the positive Z^* axis chosen along $\vec{q}$ and the positive Y^* axis chosen along $\vec{k} \wedge \vec{k}\,'$.

as proposed in [J.E. Sobczyk et al., Phys.Rev. D98 (2018) 073001]

Single pion production on the nucleon



$$\sigma \propto L^{\mu\nu}(k_1, k_2) \times H_{\mu\nu}(k, q, p_2)$$

Leptonic part (PW approximation) → known

Hadronic part → modelling effort

Exploit these facts:

- -Lepton tensor is known
- -Hadronic part is invariant under rotation along q and is the product of Hadronic current with its conjugate

→ Separate the φ* dependence

$$\frac{d\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times [A + B\cos(\phi^*)\, C\cos(2\phi^*) + D\sin(\phi^*) + E\sin(2\phi^*)]$$

A. Nikolakopoulos

Single pion production on the nucleon

Example for the A structure function:

$$A = L^{00}H_{00} + 2L^{30}H^{s}_{30} + L^{33}H_{33} + \frac{L^{11}+L^{22}}{2}(H_{11}+H_{22}) + 2iL^{12}H^{a}_{12}$$

Here the Hadron tensor depends on 3 variables:

W, Q^2, $\cos\theta_\pi^*$ and $\varphi_\pi^* = 0$

And in total one needs 15 elements of the hadron tensor

For inclusive:

Only A survives integration over pion angles:

$$\frac{d\sigma}{dQ^2 dW} = \frac{\mathcal{F}}{(2\pi)^4}\frac{k^*_\pi}{k^2_l}\times\left[L^{00}W_{CC} + 2L^{30}W_{CL} + L^{33}W_{LL} + \frac{L^{11}+L^{22}}{2}(W_T) + iL^{12}W_{T'}\right]$$

And responses depend on Q^2 and W

$$\frac{d\sigma}{dQ^2 dW d\Omega^*_\pi} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k^*_\pi}{k^2_l}\times[A + B\cos(\phi^*)\, C\cos(2\phi^*) + D\sin(\phi^*) + E\sin(2\phi^*)]$$

A. Nikolakopoulos

Implementation in Monte Carlo event generators

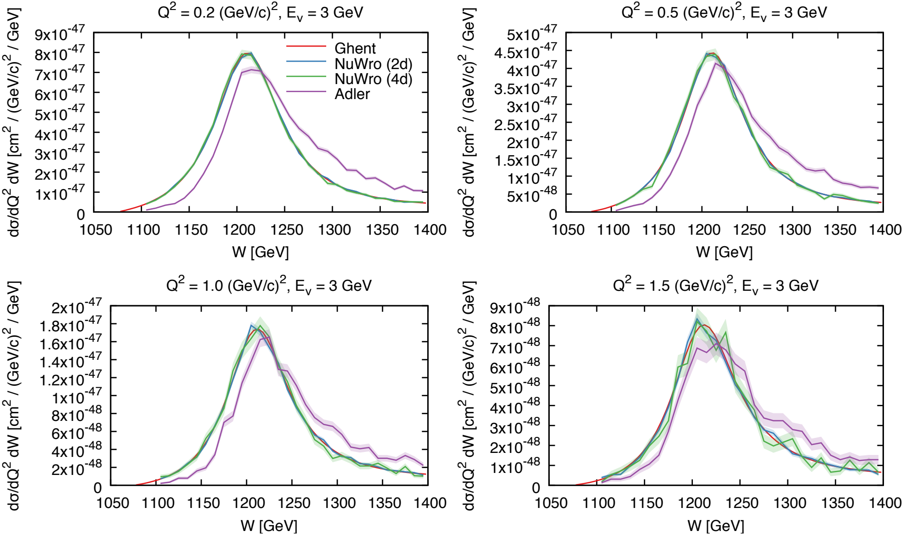
The first step:

- $\rightarrow$ tabularize 5 hadronic tensor elements as functions of (W, Q^2)
- $\rightarrow$ each time obtain the structure function A
- $\rightarrow$ sample events with double-differential cross section formula
- $\rightarrow$ add experimental pion angular distributions

The second step:

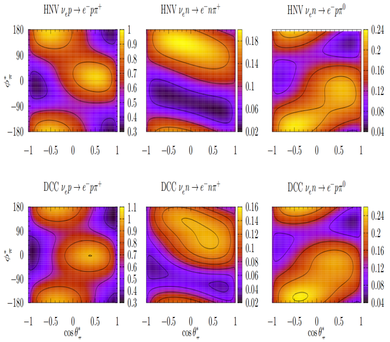
- $\rightarrow$ compute the structure functions A,B,C,D,E (W, Q^2, $\cos^* \theta_\pi$) directly from a model
- $\rightarrow$ sample events with a full 4-differential cross section formula

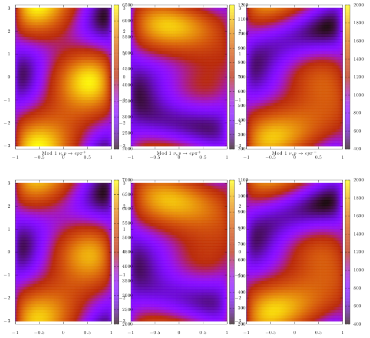
Implementation of the Ghent model



$$\nu_\mu + p \rightarrow \mu^- + p + \pi^+$$

Implementation of the Ghent model





HNV, DCC and LEM vary in structure functions, still more or less agree on angular cross section. (Around Delta peak)

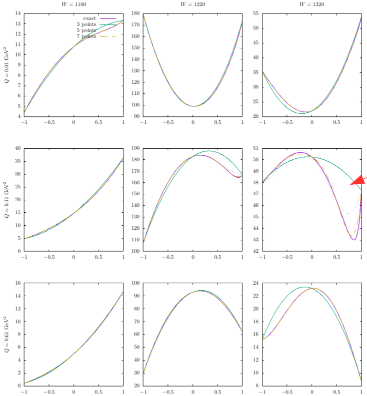
Could this influence neutrino oscillation analysis ?

A. Nikolakopoulos

Implementation of the Ghent model

given a Q2 and W, distribution of cosθ* is determined by A

$$\frac{d\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k_\pi^*}{k_l^2}\times[A + B\cos(\phi^*)\, C\cos(2\phi^*) + D\sin(\phi^*) + E\sin(2\phi^*)]$$



A is a smooth function and can usually be interpolated by a polynomial of degree 2

Calculation of A(cos) for fixed Q2 and W is very cheap

Interpolation with degree 2 polynomial means:

Cumulative distribution function

$$CDF(\cos(\theta)) = \int a_2\cos^2\theta + a1\cos\theta + a_0\, d\cos\theta$$

Is a monotonic degree 3 polynomial

→ Can be inverted analytically

→ Inversion sampling

A. Nikolakopoulos

Problems and issues