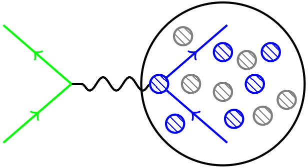

The Contribution of Two-Particle–Two-Hole Final States in Electron-Nucleus Scattering

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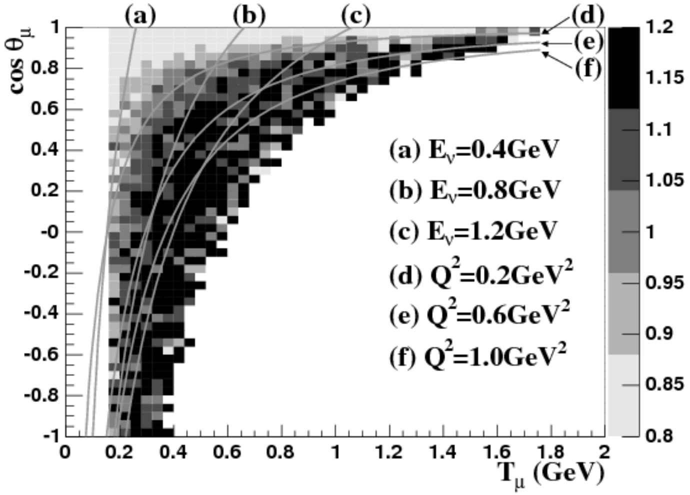
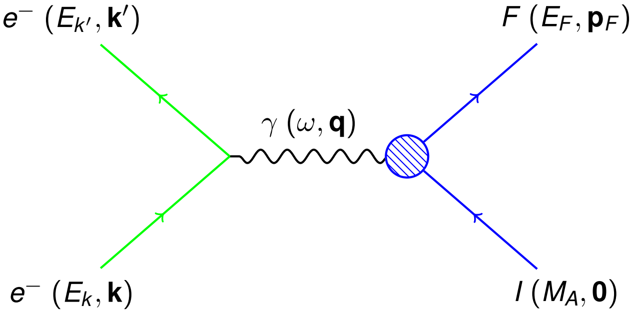
Motivation



Ratio of MiniBooNE ν_μ CCQE data versus a RFG simulation as a function of reconstructed muon angle and kinetic energy. The prediction is prior to any CCQE model adjustments. The effective axial mass extracted from the data equals $M_A^{eff} = 1.23 \pm 0.20\ \mathrm{GeV}$ [1].

Electron-nucleus interaction



Electron-nucleus interaction

Initial state:

$$|\Psi_i\rangle = |\mathbf{k}, s\rangle_e \otimes |I\rangle_A \tag{1}$$

Final state:

$$|\Psi_f\rangle = \left|\mathbf{k}', s'\right\rangle_e \otimes |F, \mathbf{p}_F\rangle_A \tag{2}$$

T-matrix element:

$$\left\langle \Psi_f \right| i\hat{T} \left| \Psi_i \right\rangle = \int d^4q \, \frac{e^2}{q^2} \, \delta_\Omega^{(4)}(q + k' - k) \, \bar{u}(\mathbf{k}', s') \, \gamma_\mu \, u(\mathbf{k}, s) \\ \times (2\pi)\delta_T(E_F - M_A + \omega) \left\langle F, \mathbf{p}_F \right| \int_V d^3\mathbf{x} \, e^{-i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^\mu(\mathbf{x}) \left| I \right\rangle . \tag{3}$$

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$$\times (2\pi)\delta_T(E_F - M_A + \omega) \left\langle F, \mathbf{p}_F \right| \int_V d^3\mathbf{x} \, e^{-i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^\mu(\mathbf{x}) \left| I \right\rangle . \tag{3}$$

Electron-nucleus interaction

Cross section:

$$\frac{d\sigma_F}{d^3\mathbf{k}' d^3\mathbf{p}_F} = \frac{1}{4}\frac{1}{E_k M_A E_{k'} E_F}\frac{\alpha^2}{q^4} L_{\mu\nu} W^{\mu\nu} \quad (4)$$

Leptonic tensor:

$$L_{\mu\nu} \equiv \frac{1}{2}\sum_{s,s'} \bar{u}(\mathbf{k}', s')\,\gamma_\mu\, u(\mathbf{k}, s)\,\bar{u}(\mathbf{k}, s)\,\gamma_\nu\, u(\mathbf{k}', s') \quad (5)$$

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Electron-nucleus interaction

Hadronic tensor:

$$W^{\mu\nu} = \sum_{\sigma_I} \left\langle F, \mathbf{p}_F \middle| \int_V d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^{\mu}(\mathbf{x}) \middle| I \right\rangle \times \left\langle I \middle| \int_V d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^{\nu\dagger}(\mathbf{x}) \middle| F, \mathbf{p}_F \right\rangle \times \frac{1}{(2\pi)^3 V} \delta(E_F - M_A - \omega) \Big|_{q=k-k'} \tag{6}$$

Electron-nucleus interaction

Inclusive cross section:

$$d\sigma = \sum_F d\sigma_F \tag{7}$$

Hadronic tensor:

$$W^{\mu\nu} = \sum_{F,\sigma_I} \int d^3\mathbf{p}_F \left\langle F, \mathbf{p}_F \middle| \int_V d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^\mu(\mathbf{x}) \middle| I \right\rangle \times \left\langle I \middle| \int_V d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^{\nu\dagger}(\mathbf{x}) \middle| F, \mathbf{p}_F \right\rangle \times \frac{1}{(2\pi)^3 V} \delta(E_F - M_A - \omega) \Big|_{q=k-k'} \tag{8}$$

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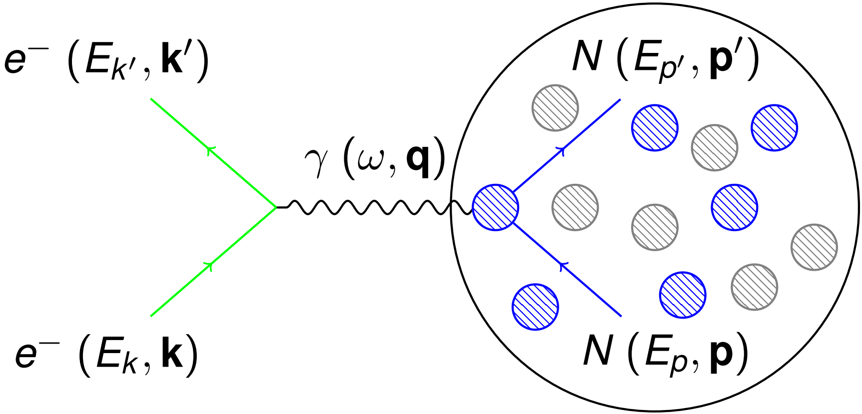
Impulse approximation (IA)



IA

One-body current:

$$\mathcal{J}^{\mu}(\mathbf{x}) \approx \sum_{N=p,n}^{A} \sum_{\sigma_{N'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_{N}}{(2\pi)^3\sqrt{2E_{N}}} \left\langle N', \mathbf{p}_{N'} \middle| j^{\mu}(\mathbf{x}) \middle| N, \mathbf{p}_N \right\rangle \times a^{\dagger}_{N'}(\mathbf{p}_{N'})\, a_N(\mathbf{p}_N)\Big|_{\tau_N=\tau_{N'}} \quad (9)$$

Matrix element:

$$\left\langle F, \mathbf{p}_F \middle| \int_V d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} \mathcal{J}^{\mu}(\mathbf{x}) \middle| I \right\rangle = $$
$$= (2\pi)^3 \sum_{N=p,n}^{A} \sum_{\sigma_{N'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_{N}}{(2\pi)^3\sqrt{2E_{N}}} \delta_V^{(3)}(\mathbf{p}_{N'} - \mathbf{p}_N - \mathbf{q}) \times \left\langle N', \mathbf{p}_{N'} \middle| j^{\mu}(0) \middle| N, \mathbf{p}_N \right\rangle \left\langle F, \mathbf{p}_F \middle| a^{\dagger}_{N'}(\mathbf{p}_{N'}) a_N(\mathbf{p}_N) \middle| I \right\rangle \quad (10)$$

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IA

Hadronic tensor:

$$
\begin{aligned}
W^{\mu\nu} = & \sum_{F,\sigma_I} \int d^3\mathbf{p}_F \, \frac{(2\pi)^3}{V} \delta(E_F - M_A - \omega) \\
& \times \sum_{N=p,n}^{A} \sum_{\sigma_{N'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_N}{(2\pi)^3\sqrt{2E_N}} \, \delta_V^{(3)}(\mathbf{p}_{N'} - \mathbf{p}_N - \mathbf{q}) \\
& \times \left\langle N', \mathbf{p}_{N'} \middle| j^\mu(0) \middle| N, \mathbf{p}_N \right\rangle \left\langle F, \mathbf{p}_F \middle| a^\dagger_{N'}(\mathbf{p}_{N'}) a_N(\mathbf{p}_N) \middle| I \right\rangle \\
& \times \sum_{M=p,n}^{A} \sum_{\sigma_{M'}} \int \frac{d^3\mathbf{p}_{M'}}{(2\pi)^3\sqrt{2E_{M'}}} \frac{d^3\mathbf{p}_M}{(2\pi)^3\sqrt{2E_M}} \, \delta_V^{(3)}(\mathbf{p}_M - \mathbf{p}_{M'} + \mathbf{q}) \\
& \times \left\langle M, \mathbf{p}_M \middle| j^{\nu\dagger}(0) \middle| M', \mathbf{p}_{M'} \right\rangle \left\langle I \middle| a^\dagger_M(\mathbf{p}_M) a_{M'}(\mathbf{p}_{M'}) \middle| F, \mathbf{p}_F \right\rangle .
\end{aligned}
\tag{11}
$$

Factorization

Elementary cross section:

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right) \sim \left|\left\langle N', \mathbf{p}_{N'} \middle| j^{\mu}(0) \middle| N, \mathbf{p}_N \right\rangle\right|^2 \tag{12}$$

Spectral function:

$$P(E, \mathbf{p}) \sim \left|\left\langle F, \mathbf{p}_F \middle| a^{\dagger}_{N'}(\mathbf{p}_{N'}) a_N(\mathbf{p}_N) \middle| I \right\rangle\right|^2 \tag{13}$$

Different matrix elements, hence no factorization in IA!

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Plane wave IA (PWIA)

Final state factorization:

$$|F, \mathbf{p}_F\rangle_A \to |X, \mathbf{p}_X\rangle \otimes |R, \mathbf{p}_R\rangle_{A-1} \tag{14}$$

The inclusive cross section:

$$d\sigma = \sum_{X,R} d\sigma_{X,R}, \tag{15}$$

$$\frac{d\sigma}{d\Omega_{k'} dE_{k'}} = \frac{1}{8(2\pi)^3} \frac{E_{k'}}{E_k} \frac{1}{M_A E_X E_R} \frac{\alpha^2}{q^4} L_{\mu\nu} W^{\mu\nu} \tag{16}$$

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\begin{aligned}
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& \times \sum_{N=p,n}^{A} \sum_{\sigma_{N'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_N}{(2\pi)^3\sqrt{2E_N}} \, \delta_V^{(3)}(\mathbf{p}_{N'} - \mathbf{p}_N - \mathbf{q}) \\
& \times \left\langle N', \mathbf{p}_{N'} \middle| j^\mu(0) \middle| N, \mathbf{p}_N \right\rangle \left\langle X, \mathbf{p}_X; R, \mathbf{p}_R \middle| a^\dagger_{N'}(\mathbf{p}_{N'}) a_N(\mathbf{p}_N) \middle| I \right\rangle \\
& \times \sum_{M=p,n}^{A} \sum_{\sigma_{M'}} \int \frac{d^3\mathbf{p}_{M'}}{(2\pi)^3\sqrt{2E_{M'}}} \frac{d^3\mathbf{p}_M}{(2\pi)^3\sqrt{2E_M}} \, \delta_V^{(3)}(\mathbf{p}_M - \mathbf{p}_{M'} + \mathbf{q}) \\
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\tag{17}
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PWIA

One-particle states annihilation:

$$\left\langle X, \mathbf{p}_X; R, \mathbf{p}_R \middle| a^\dagger_{N'}(\mathbf{p}_{N'}) a_N(\mathbf{p}_N) \middle| I \right\rangle \left\langle I \middle| a^\dagger_M(\mathbf{p}_M) a_{M'}(\mathbf{p}_{M'}) \middle| X, \mathbf{p}_X; R, \mathbf{p}_R \right\rangle$$
$$= (2\pi)^3 \sqrt{E_X} \delta^{(3)}(\mathbf{p}_X - \mathbf{p}_{N'}) \delta_{X,N'} \langle R, \mathbf{p}_R | a_N(\mathbf{p}_N) | I \rangle$$
$$\times (2\pi)^3 \sqrt{E_X} \delta^{(3)}(\mathbf{p}_X - \mathbf{p}_{M'}) \delta_{X,M'} \left\langle I \middle| a^\dagger_M(\mathbf{p}_M) \middle| R, \mathbf{p}_R \right\rangle \quad (18)$$

Identification of N and M [2]:

$$W^{\mu\nu} = \frac{1}{2} \sum_{\sigma_X, R} \sum_{N=p,n}^{A} \int d^3\mathbf{p}_X d^3\mathbf{p}\, \delta(E_F - M_A - \omega)\, \delta^{(3)}(\mathbf{p}_X - \mathbf{p} - \mathbf{q})$$
$$\times \frac{1}{(2\pi)^6 2E_N} \left| \langle X, \mathbf{p}_X | j^\mu(0) | N, \mathbf{p} \rangle \right|^2 \left| \langle R, -\mathbf{p} | a_N(\mathbf{p}) | I \rangle \right|^2 \quad (19)$$

PWIA

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PWIA

Spectral function:

$$P_N(\mathbf{p}, E) = \frac{1}{(2\pi)^6 2E_N} \sum_R |\langle R, -\mathbf{p}|a_N(\mathbf{p})|I\rangle|^2 \, \delta(E - M + M_A - E_R) \quad (20)$$

Hadronic tensor:

$$W^{\mu\nu} = \sum_{\sigma_X} \sum_{N=p,n}^{A} \int d^3\mathbf{p}_X d^3\mathbf{p} dE \, P_N(\mathbf{p}, E) \times |\langle X, \mathbf{p}_X|j^\mu(0)|N, \mathbf{p}\rangle|^2 \, \delta(M - E - E_X + \omega) \, \delta^{(3)}(\mathbf{p}_X - \mathbf{p} - \mathbf{q}), \quad (21)$$

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PWIA

Elementary hadronic tensor:

$$\omega_N^{\mu\nu} \equiv \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \left|\left\langle N', \mathbf{p}' \middle| j^\mu(0) \middle| N, \mathbf{p} \right\rangle\right|^2 \delta(M - E - E_{p'} + \omega)\, \delta^{(3)}(\mathbf{p}' - \mathbf{p} - \mathbf{q}) \tag{22}$$

Effective energy transfer:

$$\tilde{\omega} \equiv \omega - B = \omega + M - E - \sqrt{\mathbf{p}^2 + M^2} \tag{23}$$

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \tag{24}$$

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Factorized cross section

$$W^{\mu\nu} = \int d^3\mathbf{p}' d^3\mathbf{p} dE \, \left(Z P_p(\mathbf{p}, E) \, \omega_p^{\mu\nu} + (A - Z) P_n(\mathbf{p}, E) \, \omega_n^{\mu\nu} \right) \quad (26)$$

Integration:

$$d\sigma = d\Omega_{k'} dE_{k'} d^3\mathbf{p}' d^3\mathbf{p} dE ... \quad (27)$$

$$P(\mathbf{p}, E) : \delta(...) \quad (28)$$

$$\omega^{\mu\nu} : \delta^{(4)}(...) \quad (29)$$

$$\rightarrow d\sigma = d\Omega_{k'} d^3\mathbf{p} dE ... \quad (30)$$

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Integration:

$$d\sigma = d\Omega_{k'} dE_{k'} d^3\mathbf{p}' d^3\mathbf{p} dE ... \tag{27}$$

$$P(\mathbf{p}, E) : \delta(...) \tag{28}$$

$$\omega^{\mu\nu} : \delta^{(4)}(...) \tag{29}$$

$$\rightarrow d\sigma = d\Omega_{k'} d^3\mathbf{p} dE ... \tag{30}$$

Factorized cross section

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)_A = \int d^3\mathbf{p}dE\,\chi\left(ZP_p(\mathbf{p},E)\left(\frac{d\sigma_p}{d\Omega_{k'}}\right) + (A-Z)P_n(\mathbf{p},E)\left(\frac{d\sigma_n}{d\Omega_{k'}}\right)\right) \tag{31}$$

Kinematical factor:

$$\chi = \frac{1}{2(2\pi)^3}\frac{ME_p}{M_A^2 E_R} \tag{32}$$

Elementary cross section:

$$\left(\frac{d\sigma_N}{d\Omega_{k'}}\right) = \frac{1}{4}\frac{E_{k'}^2}{E_k^2}\frac{1}{ME_p}\frac{\alpha^2}{q^4}L_{\mu\nu}\omega_N^{\mu\nu} \tag{33}$$

J.A. Caballero et al. [3]

$$\frac{d^5\sigma}{d\Omega_{k'}dE_{k'}d\Omega_{p'}} = \frac{2\alpha^2}{q^4}\left(\frac{E_{k'}}{E_k}\right)\frac{|\mathbf{p}'|MM_R}{M_A f_{rec}} 2\overline{\sum}|\mathcal{M}|^2 \quad (34)$$

where

$$\mathcal{M} = j^e_\mu \mathcal{J}^\mu_N \quad (35)$$

$$j^e_\mu = \bar{u}_{\sigma_{k'}}(\mathbf{k}')\gamma^\mu u_{\sigma_k}(\mathbf{k}) \quad (36)$$

$$\mathcal{J}^\mu_N = \bar{u}_{\sigma_{p'}}(\mathbf{p}')\hat{\mathcal{J}}^\mu \Psi^{m_b}_b(\mathbf{p}) \quad (37)$$

Using completeness:

$$\mathcal{J}^\mu_N = \bar{u}_{\sigma_{p'}}(\mathbf{p}')\hat{\mathcal{J}}^\mu u_{\sigma_p}(\mathbf{p})[\bar{u}_{\sigma_p}(\mathbf{p})\Psi^{m_b}_b(\mathbf{p})] \quad (38)$$

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Specific nucleon solution

Initial state factorization (mean-field):

$$|I\rangle \rightarrow \sum_b \int d^3\mathbf{p}_b \left(|R_b, -\mathbf{p}_b\rangle \otimes |b, \mathbf{p}_b\rangle\right) \alpha_b(\mathbf{p}_b) \tag{40}$$

Matrix element:

$$\langle R, -\mathbf{p}|a_N(\mathbf{p})|I\rangle = \sum_b (2\pi)^3 \sqrt{2E_N}\, \delta_{R,R_b}\, \delta_{N,b}\, \alpha_b(\mathbf{p}) \tag{41}$$

Integration:

$$d\sigma = d\Omega_{k'} dE_{k'} d^3\mathbf{p}' d^3\mathbf{p} dE ... \tag{42}$$

$$P(\mathbf{p}, E) : \delta(...) \tag{43}$$

$$\omega^{\mu\nu} : \delta^{(4)}(...) \tag{44}$$

$$\rightarrow d\sigma = d\Omega_{k'} dE_{k'} d\Omega_{p'} ... \tag{45}$$

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Specific nucleon solution

Cross section:

$$\left(\frac{d\sigma_b}{d\Omega_{k'}dE_{k'}d\Omega_{p'}}\right)_A = \sum_b \chi \left(\frac{d\sigma}{d\Omega_{k'}}\right) \alpha_b^2(\mathbf{p}) \tag{46}$$

where

$$\chi = \frac{1}{(2\pi)^3} \frac{E_k}{E_{k'}} \frac{ME_p|\mathbf{p}'|}{M_A E_R} \tag{47}$$

J.A. Caballero et al. [3] - Relativistic PWIA (RPWIA)

Completeness:

$$\sum_s u_\alpha(\mathbf{p}, s)\bar{u}_\beta(\mathbf{p}, s) - v_\alpha(\mathbf{p}, s)\bar{v}_\beta(\mathbf{p} = \delta_{\alpha\beta} \tag{48}$$

Now

$$J_N^\mu = J_u^\mu - J_v^\mu \tag{49}$$

where

$$\mathcal{J}_u^\mu = \bar{u}_{\sigma_{p'}}(\mathbf{p}')\hat{\mathcal{J}}^\mu u_{\sigma_p}(\mathbf{p})[\bar{u}_{\sigma_p}(\mathbf{p})\Psi_b^{m_b}(\mathbf{p})] \tag{50}$$

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and

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RWPIA

One-body current:

$$\mathcal{J}^{\mu}(\mathbf{x}) \approx \sum_{N=p,n}^{A} \sum_{\sigma_{N'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_N}{(2\pi)^3\sqrt{2E_N}}\Bigg|_{\tau_N=\tau_{N'}} \times \Big(\big\langle N', \mathbf{p}'_N \big| j^{\mu}(\mathbf{x}) \big| N, \mathbf{p}_N \big\rangle \, a^{\dagger}_{N'}(\mathbf{p}_{N'}) \, a_N(\mathbf{p}_N) - \big\langle N', \mathbf{p}'_N; \bar{N}, \mathbf{p}_N \big| j^{\mu}(\mathbf{x}) \big| \varnothing \big\rangle \, a^{\dagger}_{N'}(\mathbf{p}_{N'}) \, b^{\dagger}_N(\mathbf{p}_N) \Big) \tag{54}$$

Three components:

$$W^{\mu\nu} = \mathcal{W}^{\mu\nu} + \mathcal{Z}^{\mu\nu} + \mathcal{N}^{\mu\nu} \tag{55}$$

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$$\times \Big(\big\langle N', \mathbf{p}'_N \big| j^{\mu}(\mathbf{x}) \big| N, \mathbf{p}_N \big\rangle \, a^{\dagger}_{N'}(\mathbf{p}_{N'}) \, a_N(\mathbf{p}_N)$$
$$- \big\langle N', \mathbf{p}'_N; \bar{N}, \mathbf{p}_N \big| j^{\mu}(\mathbf{x}) \big| \varnothing \big\rangle \, a^{\dagger}_{N'}(\mathbf{p}_{N'}) \, b^{\dagger}_N(\mathbf{p}_N) \Big) \tag{54}$$

Three components:

$$W^{\mu\nu} = \mathcal{W}^{\mu\nu} + \mathcal{Z}^{\mu\nu} + \mathcal{N}^{\mu\nu} \tag{55}$$

RWPIA

Positive energy:

$$\mathcal{W}^{\mu\nu} = \frac{1}{2} \sum_{\sigma_X} \sum_{N=p,n}^{A} \int d^3\mathbf{p}_X d^3\mathbf{p} dE \; P_N(\mathbf{p}, E) \; \left|\langle X, \mathbf{p}_X | j^\mu(0) | N, \mathbf{p}\rangle\right|^2 \\ \times \delta(M - E - E_X + \omega) \, \delta^{(3)}(\mathbf{p}_X - \mathbf{p} - \mathbf{q}) \tag{56}$$

where

$$P_N(\mathbf{p}, E) = \frac{1}{(2\pi)^6 2E_N} \sum_R \left|\langle R, -\mathbf{p} | a_N(\mathbf{p}) | I\rangle\right|^2 \; \delta(E - M + E_I - E_R) \tag{57}$$

RWPIA

Negative energy:

$$\mathcal{Z}_N^{\mu\nu} = \frac{1}{2}\sum_{\sigma_X}\sum_{N=p,n}^{A}\int d^3\mathbf{p}_X d^3\mathbf{p} dE\, N_N(\mathbf{p},E)\, \left|\left\langle X,\mathbf{p}_X;\bar{N},\mathbf{p}\middle|j^\mu(0)\middle|\varnothing\right\rangle\right|^2$$
$$\times\, \delta(M-E-E_X+\omega)\,\delta^{(3)}(\mathbf{p}_X+\mathbf{p}-\mathbf{q}) \tag{58}$$

where

$$N_N(\mathbf{p},E) = \frac{1}{(2\pi)^6 2E_N}\sum_R \left|\left\langle R,\mathbf{p}\middle|b_N^\dagger(\mathbf{p})\middle|I\right\rangle\right|^2\, \delta(E-M+E_I-E_R) \tag{59}$$

RWPIA

Crossed:

$$\begin{aligned}\mathcal{N}^{\mu\nu} = -\frac{1}{2}\sum_{\sigma_X,R}\sum_{N=p,n}^{A}\frac{1}{(2\pi)^6 2E_N}\int d^3\mathbf{p}_X d^3\mathbf{p}\,\delta(E_F - M_A - \omega)\,\frac{(2\pi)^3}{V}\\
\times\Big[\delta_V^{(3)}(\mathbf{p}_X - \mathbf{p} - \mathbf{q})\,\delta_V^{(3)}(\mathbf{p}_X + \mathbf{p} - \mathbf{q})\\
\times\langle X,\mathbf{p}_X|j^\mu(0)|N,\mathbf{p}\rangle\left\langle\varnothing\middle|j^{\dagger\nu}(0)\middle|X,\mathbf{p}_X;\bar{N},\mathbf{p}\right\rangle\\
\times\langle R,-\mathbf{p}|a_N(\mathbf{p})|I\rangle\,\langle I|b_N(\mathbf{p})|R,\mathbf{p}\rangle\\
+\,\delta_V^{(3)}(\mathbf{p}_X + \mathbf{p} - \mathbf{q})\,\delta_V^{(3)}(\mathbf{p}_X - \mathbf{p} - \mathbf{q})\\
\times\left\langle X,\mathbf{p}_X;\bar{N},\mathbf{p}\middle|j^\mu(0)\middle|\varnothing\right\rangle\left\langle N,\mathbf{p}\middle|j^{\nu\dagger}(0)\middle|X,\mathbf{p}_X\right\rangle\\
\times\left\langle R,\mathbf{p}\middle|b_N^\dagger(\mathbf{p})\middle|I\right\rangle\left\langle I\middle|a_N^\dagger(\mathbf{p})\middle|R,-\mathbf{p}\right\rangle\Big]\end{aligned}\tag{60}$$

Specific nucleon solution

$$|R, \mathbf{p}\rangle \to \sum_b \int d^3\mathbf{p}_b \left(|I_b, \mathbf{p} - \mathbf{p}_b\rangle \otimes \left|\bar{b}, \mathbf{p}_b\right\rangle\right) \beta_b(\mathbf{p}_b) \tag{61}$$

$$\left\langle R, \mathbf{p} \middle| b_N^\dagger(\mathbf{p}) \middle| I \right\rangle = \sum_b (2\pi)^3 \sqrt{2E_N}\, \delta_{I,I_b}\, \delta_{N,b}\, \beta_b(\mathbf{p}) \tag{62}$$

Negative energy spectral function:

$$\sum_{N=p,n}^{A} N_N(\mathbf{p}, E) = \sum_b \beta_b^2(\mathbf{p})\, \delta(E - M + M_A - E_R) \tag{63}$$

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Specific nucleon solution

Crossed term:

$$\begin{aligned}
\mathcal{N}^{\mu\nu} = & -\int d^3\mathbf{p}\prime d^3\mathbf{p} dE \, \alpha_b(\mathbf{0})\beta_b(\mathbf{0})\delta(E - M + M_A - E_R) \\
& \times \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \delta(M - E - E_{N'} + \omega) \, \delta^{(3)}(\mathbf{p}' - \mathbf{q}) \Big|_{\mathbf{p}=0} \\
& \times \Big[\left\langle N', \mathbf{p}' \middle| j^{\mu}(0) \middle| N, \mathbf{0} \right\rangle \left\langle \varnothing \middle| j^{\dagger\nu}(0) \middle| N', \mathbf{p}'; \bar{N}, \mathbf{0} \right\rangle \\
& + \left\langle N', \mathbf{p}'; \bar{N}, \mathbf{0} \middle| j^{\mu}(0) \middle| \varnothing \right\rangle \left\langle N, \mathbf{0} \middle| j^{\nu\dagger}(0) \middle| N', \mathbf{p}' \right\rangle \Big]
\end{aligned} \tag{64}$$

Specific nucleon solution

Elementary hadronic tensors:

$$\omega^{\mu\nu} = \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \left| \left\langle N', \mathbf{p}' \middle| j^\mu(0) \middle| N, \mathbf{p} \right\rangle \right|^2 \tag{65}$$

$$\zeta^{\mu\nu} = \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \left| \left\langle X, \mathbf{p}_X; \bar{N}, \mathbf{p} \middle| j^\mu(0) \middle| \varnothing \right\rangle \right|^2 \tag{66}$$

$$\begin{aligned} \eta^{\mu\nu} = \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \Big[& \left\langle N', \mathbf{p}' \middle| j^\mu(0) \middle| N, \mathbf{0} \right\rangle \left\langle \varnothing \middle| j^{\dagger\nu}(0) \middle| N', \mathbf{p}'; \bar{N}, \mathbf{0} \right\rangle \\ & + \left\langle N', \mathbf{p}'; \bar{N}, \mathbf{0} \middle| j^\mu(0) \middle| \varnothing \right\rangle \left\langle N, \mathbf{0} \middle| j^{\nu\dagger}(0) \middle| N', \mathbf{p}' \right\rangle \Big] \end{aligned} \tag{67}$$

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$$\omega^{\mu\nu} = \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \left| \left\langle N', \mathbf{p}' \middle| j^{\mu}(0) \middle| N, \mathbf{p} \right\rangle \right|^2 \tag{65}$$

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$$\begin{aligned} \eta^{\mu\nu} = \frac{1}{2} \sum_{\sigma_{N'},\sigma_N} \Big[& \left\langle N', \mathbf{p}' \middle| j^{\mu}(0) \middle| N, \mathbf{0} \right\rangle \left\langle \varnothing \middle| j^{\dagger\nu}(0) \middle| N', \mathbf{p}'; \bar{N}, \mathbf{0} \right\rangle \\ & + \left\langle N', \mathbf{p}'; \bar{N}, \mathbf{0} \middle| j^{\mu}(0) \middle| \varnothing \right\rangle \left\langle N, \mathbf{0} \middle| j^{\nu\dagger}(0) \middle| N', \mathbf{p}' \right\rangle \Big] \end{aligned} \tag{67}$$

Specific nucleon solution

Cross section:

$$\left(\frac{d\sigma_b}{d\Omega_{k'} dE_{k'} d\Omega_{p'}}\right)_A = \chi\left[\left(\frac{d\sigma_P}{d\Omega_{k'}}\right)\alpha_b^2(\mathbf{p}) + \left(\frac{d\sigma_N}{d\Omega_{k'}}\right)\beta_b^2(\mathbf{p}) + \left(\frac{d\sigma_C}{d\Omega_{k'}}\right)\alpha_b(\mathbf{0})\beta_b(\mathbf{0})\right] \tag{68}$$

where

$$\chi = \frac{1}{(2\pi)^3}\frac{E_k}{E_{k'}}\frac{ME_p|\mathbf{p}'|}{M_A E_R} \tag{69}$$

2p2h

Neglecting FSI:

$$d\sigma_A = d\sigma_{1p1h} + d\sigma_{2p2h} \propto L_{\mu\nu}(W^{\mu\nu}_{1p1h} + W^{\mu\nu}_{2p2h}) \tag{70}$$

One- and two-body current:

$$\mathcal{J}^\mu(\mathbf{x}) \approx \mathcal{J}^\mu_1(\mathbf{x}) + \mathcal{J}^\mu_2(\mathbf{x}) \tag{71}$$

$$W^{\mu\nu}_{2p2h} = W^{\mu\nu}_{11} + W^{\mu\nu}_{12} + W^{\mu\nu}_{22} \tag{72}$$

Term 11: we wish to have P_{1p2h}
Term 12: we wish to have P_{2h}

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Term 11: we wish to have P_{1p2h}
Term 12: we wish to have P_{2h}

2p2h

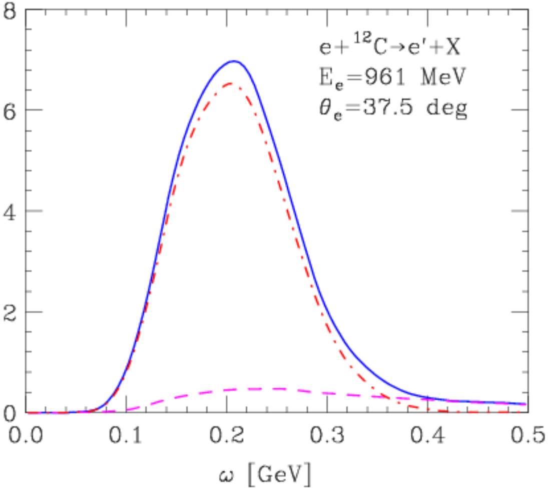
Two-body current:

$$\mathcal{J}_2^{\mu}(\mathbf{x}) = \sum_{N=p,n}^{A} \sum_{M=p,n}^{A-1} \sum_{\sigma_{N'},\sigma_{M'}} \int \frac{d^3\mathbf{p}_{N'}}{(2\pi)^3\sqrt{2E_{N'}}} \frac{d^3\mathbf{p}_N}{(2\pi)^3\sqrt{2E_N}}$$
$$\times \int \frac{d^3\mathbf{p}_{M'}}{(2\pi)^3\sqrt{2E_{M'}}} \frac{d^3\mathbf{p}_M}{(2\pi)^3\sqrt{2E_M}} \quad (73)$$
$$\times \left\langle N', \mathbf{p}_{N'}; M', \mathbf{p}_{M'} \middle| j^{\mu}(\mathbf{x}) \middle| N, \mathbf{p}_N; M, \mathbf{p}_M \right\rangle$$
$$\times\, a^{\dagger}_{N'}(\mathbf{p}_{N'})\, a^{\dagger}_{M'}(\mathbf{p}_{M'})\, a_N(\mathbf{p}_N) a_M(\mathbf{p}_M)$$

2p2h

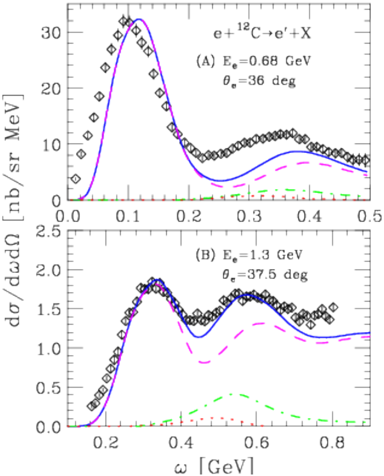
$$
\begin{aligned}
W_{22}^{\mu\nu} = & \sum_{\sigma_X,\sigma_Y,R,\sigma_I} \int d^3\mathbf{p}_X d^3\mathbf{p}_Y d^3\mathbf{p}_R \, \frac{(2\pi)^3}{V} \delta(E_F - M_A - \omega) \\
& \times \sum_{N=p,n}^{A} \sum_{M=p,n}^{A-1} \int \frac{d^3\mathbf{p}_N}{(2\pi)^3\sqrt{2E_N}} \frac{d^3\mathbf{p}_M}{(2\pi)^3\sqrt{2E_M}} \, \delta_V^{(3)}(\mathbf{p}_X + \mathbf{p}_Y - \mathbf{p}_N - \mathbf{p}_M - \mathbf{q}) \\
& \times \langle X, \mathbf{p}_X; Y, \mathbf{p}_Y | j^\mu(0) | N, \mathbf{p}_N; M, \mathbf{p}_M \rangle \, \langle R, \mathbf{p}_R | a_N(\mathbf{p}_N) a_M(\mathbf{p}_M) | I \rangle \\
& \times \sum_{O=p,n}^{A} \sum_{P=p,n}^{A-1} \int \frac{d^3\mathbf{p}_O}{(2\pi)^3\sqrt{2E_O}} \frac{d^3\mathbf{p}_P}{(2\pi)^3\sqrt{2E_P}} \, \delta_V^{(3)}(\mathbf{p}_O + \mathbf{p}_P - \mathbf{p}_X - \mathbf{p}_Y + \mathbf{q}) \\
& \times \left\langle O, \mathbf{p}_O; P, \mathbf{p}_P \middle| j^{\nu\dagger}(0) \middle| X, \mathbf{p}_X; Y, \mathbf{p}_Y \right\rangle \left\langle I \middle| a_P^\dagger(\mathbf{p}_P) a_O^\dagger(\mathbf{p}_O) \middle| R, \mathbf{p}_R \right\rangle .
\end{aligned}
\tag{74}
$$

Factorization ansatz [4]



Relative momentum distribution of a nucleon pair in isospin symmetric nuclear matter at equilibrium density.

Factorization ansatz [5]



Double differential cross section of the process $e + C \rightarrow e' + X$. The solid line shows the result of the full calculation, while the dashed line has been obtained including the one-body current only. The contributions arising from the two-nucleon current are illustrated by the dot-dash and dotted lines, corresponding to the pure two-body current transition probability and to the interference term.

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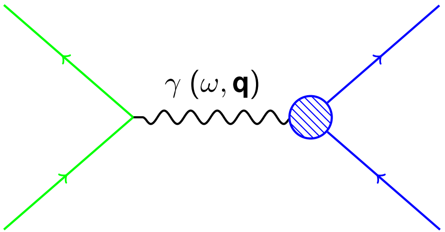
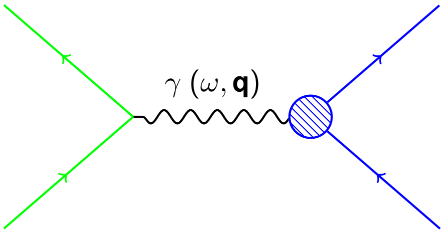
Backup slides

One-photon exchange approximation



$$\left\langle \Psi_f \middle| i\hat{T} \middle| \Psi_i \right\rangle = -i \int \frac{d^4q}{(2\pi)^4} \left(\frac{-ig^{\mu\nu}}{q^2} \right) \times \left(-ie \left\langle \mathbf{k}', s' \middle| \int_\Omega d^4x \, e^{-iq\cdot x} j_\mu(x) \middle| \mathbf{k}, s \right\rangle \right) \times \left(-ie \left\langle \mathbf{p}', r' \middle| \int_\Omega d^4y \, e^{iq\cdot y} \mathcal{J}_\nu(y) \middle| \mathbf{p}, r \right\rangle \right) \tag{75}$$

One-photon exchange approximation



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Electron-nucleon interaction

General cross section formula reads

$$d\sigma = \frac{1}{2E_k 2E_p} \frac{d^3\mathbf{k}'}{2(2\pi)^3 E_{k'}} \frac{d^3\mathbf{p}'}{2(2\pi)^3 E_{p'}} \frac{1}{\Omega} \left| \left\langle \Psi_f \right| i\hat{T} \left| \Psi_i \right\rangle \right|^2 . \tag{76}$$

Taking four-momentum eigenstates

$$\begin{aligned} \left\langle \Psi_f \right| i\hat{T} \left| \Psi_i \right\rangle &= \int \frac{d^4 q}{(2\pi)^4} \frac{e^2}{q^2} \\ &\times (2\pi)^4 \delta_\Omega^{(4)}(q + k' - k) \left\langle \mathbf{k}', s' \middle| j_\mu(0) \middle| \mathbf{k}, s \right\rangle \\ &\times (2\pi)^4 \delta_\Omega^{(4)}(q - p' + p) \left\langle \mathbf{p}', r' \middle| \mathcal{J}^\mu(0) \middle| \mathbf{p}, r \right\rangle \\ &= (2\pi)^4 \frac{e^2}{q^2} \delta_\Omega^{(4)}(k - k' - p' + p) \\ &\times \left\langle \mathbf{k}', s' \middle| j_\mu(0) \middle| \mathbf{k}, s \right\rangle \left\langle \mathbf{p}', r' \middle| \mathcal{J}^\mu(0) \middle| \mathbf{p}, r \right\rangle . \end{aligned} \tag{77}$$

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Electron-nucleon interaction

The cross section reads

$$\frac{d\sigma}{d^3\mathbf{k}' d^3\mathbf{p}'} = \frac{1}{4}\frac{1}{E_k E_p E_{k'} E_{p'}}\frac{\alpha^2}{q^4} L_{\mu\nu} W^{\mu\nu}, \tag{78}$$

where the following structures have been used:

- the leptonic tensor:

$$L_{\mu\nu} \equiv \frac{1}{2}\sum_{s,s'} \left\langle \mathbf{k}', s' \middle| j_\mu(0) \middle| \mathbf{k}, s \right\rangle \left\langle \mathbf{k}', s' \middle| j_\nu(0) \middle| \mathbf{k}, s \right\rangle^*, \tag{79}$$

- the hadronic tensor:

$$W^{\mu\nu} \equiv \frac{1}{2}\sum_{r,r'} \left\langle \mathbf{p}', r' \middle| \mathcal{J}^\mu(0) \middle| \mathbf{p}, r \right\rangle \left\langle \mathbf{p}', r' \middle| \mathcal{J}^\nu(0) \middle| \mathbf{p}, r \right\rangle^* \times \delta^{(4)}(p' - p - q)\Big|_{q=k-k'}. \tag{80}$$

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Electron-nucleon interaction

Integrating out the delta function, one obatains

$$\frac{d\sigma}{d\Omega_{k'}} = \frac{1}{4}\frac{E_{k'}^2}{E_k^2}\frac{1}{E_p^2}\frac{\alpha^2}{q^4}L_{\mu\nu}H^{\mu\nu}, \quad (81)$$

where

$$W^{\mu\nu} = H^{\mu\nu}\delta^{(4)}(p' - p - q). \quad (82)$$

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Electromagnetic form factors

An effective hadronic vertex

$$\left\langle \mathbf{p}', r' \middle| \mathcal{J}^{\mu}(0) \middle| \mathbf{p}, r \right\rangle = \bar{u}(\mathbf{p}', r')\, \Gamma^{\mu}(q^2)\, u(\mathbf{p}, r), \tag{83}$$

where

$$\Gamma^{\mu}(q^2) = \gamma^{\mu} F_1(q^2) + \frac{i}{2M}\sigma^{\mu\alpha} q_{\alpha} F_2(q^2). \tag{84}$$

The final result

$$\frac{d\sigma}{d\Omega_{k'}} = \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{\mathrm{Mott}} \left[\left(F_1^2 - \frac{q^2}{4M^2}F_2^2\right) - (F_1 + F_2)^2 \frac{q^2}{2M^2} \tan^2\frac{\theta}{2}\right], \tag{85}$$

where

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{\mathrm{Mott}} = \frac{\alpha^2 E_{k'} \cos^2\frac{\theta}{2}}{4E_k^3 \sin^4\frac{\theta}{2}}. \tag{86}$$

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