Constructing Efficient Monte Carlo Generators

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Outline of the lecture

Part I

- Introduction
- General neutrino Monte Carlo scheme
- General optimization tricks

Part II

- Selected interaction channels
- All-in-one example: intranuclear cascade.
- Summary

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Part I

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Introduction

Purpose of MC simulations

- In HEP experiments: simulation of particle interactions.
- Monte Carlo: statistical description and tool to understand your experiment with all its systematic and statistical errors.
- Lots of input and dependencies:

theoretical models, experimental data, engineering knowledge etc.

The transverse response:

$$R_T = \frac{-2}{\pi}\Im\Pi^{11}_{RFG}(q) = \frac{\Omega}{2\pi^2 q}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{E_{min}}^{E_F} dE(p)A^{11}_N = \tag{4.86}$$
$$= \frac{\Omega}{2\pi^2 q}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{E_{min}}^{E_F} dE(p)(2p_1^2)\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) - \frac{1}{2}\tilde{q}_\mu^2(F_1+F_2)^2.$$

The p_1^2:

$$p_1^2 = p^2\sin^2\Theta_p\sin^2\phi_p = \left(E(p)^2 - M^2 - \frac{(2E(p)\tilde{q}^0 + \tilde{q}_\mu^2)^2}{4q^2}\right) = \tag{4.87}$$
$$= p^2\sin^2\Theta_p\sin^2\phi_p = \left(-E(p)^2\frac{\tilde{q}_\mu^2}{q^2} - E(p)\tilde{q}^0\frac{\tilde{q}_\mu^2}{q^2} - M^2 - \frac{\tilde{q}_\mu^4}{4q^2}\right)\sin^2\phi_p.$$

And $\int_0^{2\pi}\frac{d\phi}{2\pi}\sin^2\phi = \frac{1}{2}$, thus

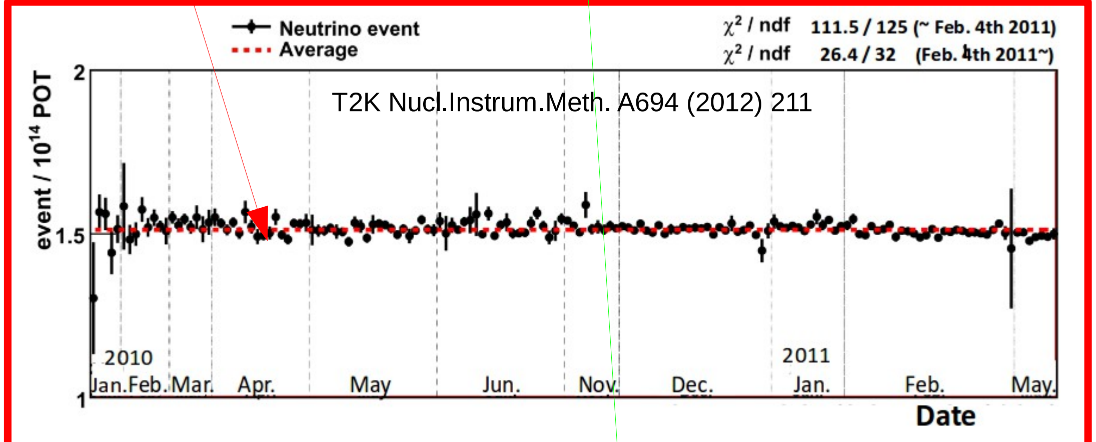
$$R_T = -\frac{\Omega}{2\pi^2 q}\left[\left(\frac{E(p)^3}{3}\frac{\tilde{q}_\mu^2}{q^2} + \frac{E(p)^2\tilde{q}^0\tilde{q}_\mu^2}{2q^2} + E(p)(M^2 + \frac{\tilde{q}_\mu^4}{4q^2})\right)\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) + \right. \tag{4.88}$$
$$\left. + \frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}}^{E_F}.$$

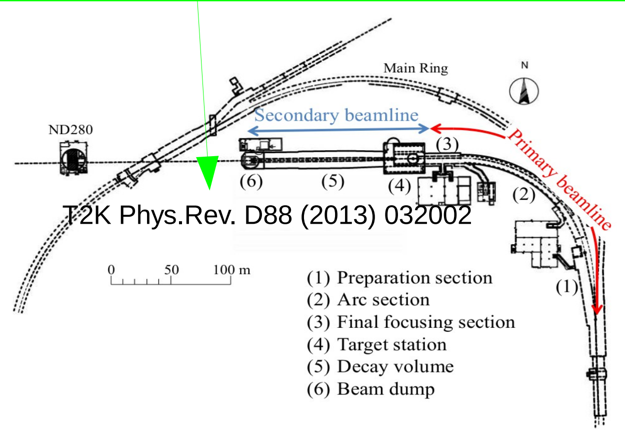
The cross section (per nucleon) in the FG case:

$$\frac{d\sigma}{d\Omega dE'} = \frac{3\sigma_{Mott}}{4k_F^3 q}\left\{\frac{q_\mu^4}{\boldsymbol{q}^4}\left[(\frac{2}{3}E(p)^3 + \tilde{q}^0E(p)^2)\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) + \right.\right. \tag{4.89}$$
$$\left. + \frac{1}{2}\tilde{q}_\mu^2E(p)(F_1+F_2)^2 - \tilde{q^0}^2E(p)\left(F_1F_2 + \frac{1}{2}(1 + \frac{\tilde{q}_\mu^2}{4M^2})F_2^2\right)\right]_{E_{min}}^{E_F} +$$
$$- \left(-\frac{q_\mu^2}{\boldsymbol{q}^2} + 2\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\right)\left[\left(\frac{E(p)^3}{3}\frac{\tilde{q}_\mu^2}{q^2} + \frac{E(p)^2\tilde{q}^0\tilde{q}_\mu^2}{2q^2} + E(p)(M^2 + \frac{\tilde{q}_\mu^4}{4q^2})\right)\right.$$
$$\left.\left.\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) + \frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}}^{E_F}\right\}.$$

The cross section (total for protons/neutrons) in the LFG case:

$$\frac{d\sigma}{d\Omega dE'} = \frac{\sigma_{Mott}}{\pi q}\int r^2dr\left\{\frac{q_\mu^4}{\boldsymbol{q}^4}\left[(\frac{2}{3}E(p)^3 + \tilde{q}^0E(p)^2)\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) + \right.\right. \tag{4.90}$$
$$\left. + \frac{1}{2}\tilde{q}_\mu^2E(p)(F_1+F_2)^2 - \tilde{q^0}^2E(p)\left(F_1F_2 + \frac{1}{2}(1 + \frac{\tilde{q}_\mu^2}{4M^2})F_2^2\right)\right]_{E_{min}}^{E_F} +$$
$$- \left(-\frac{q_\mu^2}{\boldsymbol{q}^2} + 2\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\right)\left[\left(\frac{E(p)^3}{3}\frac{\tilde{q}_\mu^2}{q^2} + \frac{E(p)^2\tilde{q}^0\tilde{q}_\mu^2}{2q^2} + E(p)(M^2 + \frac{\tilde{q}_\mu^4}{4q^2})\right)\right.$$
$$\left.\left.\left(F_1^2 - \frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right) + \frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}(r)}^{E_F(r)}\right\}.$$





Purpose of MC simulations

- Shortly: how to put it all together and get from here:

The transverse response:

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$$+\frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}}^{E_F}.$$

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$$+\frac{1}{2}\tilde{q}_\mu^2E(p)(F_1+F_2)^2-\tilde{q}^{0\,2}E(p)\left(F_1F_2+\frac{1}{2}(1+\frac{\tilde{q}_\mu^2}{4M^2})F_2^2\right)\Big]_{E_{min}}^{E_F}+$$
$$-\left(-\frac{q_\mu^2}{\boldsymbol{q}^2}+2\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\right)\left[\left(\frac{E(p)^3}{3}\frac{\tilde{q}_\mu^2}{q^2}+\frac{E(p)^2\tilde{q}^0\tilde{q}_\mu^2}{2q^2}+E(p)(M^2+\frac{\tilde{q}_\mu^4}{4q^2})\right)\right.$$
$$\left.\left(F_1^2-\frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right)+\frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}}^{E_F}\Big\}.$$

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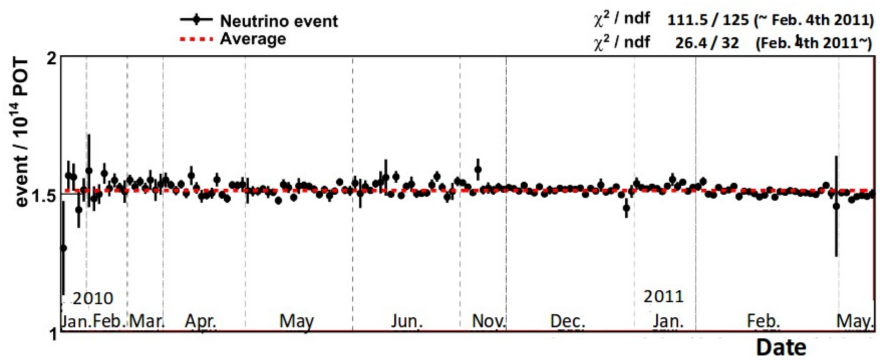
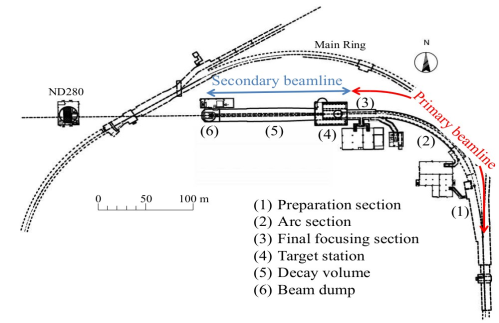
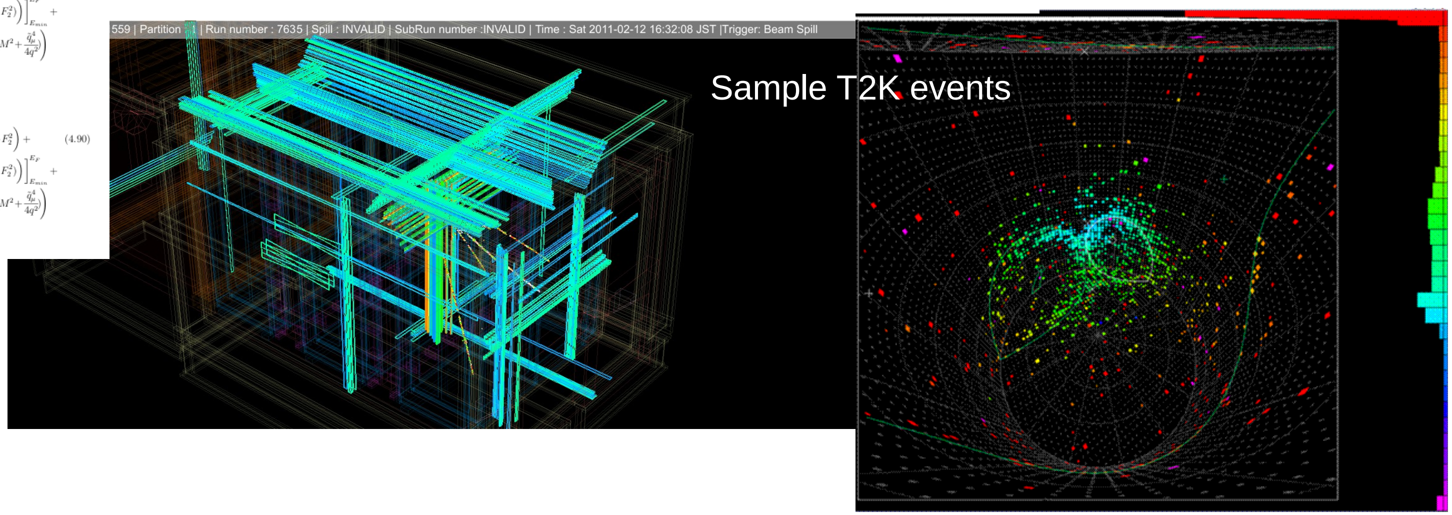
$$-\left(-\frac{q_\mu^2}{\boldsymbol{q}^2}+2\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\right)\left[\left(\frac{E(p)^3}{3}\frac{\tilde{q}_\mu^2}{q^2}+\frac{E(p)^2\tilde{q}^0\tilde{q}_\mu^2}{2q^2}+E(p)(M^2+\frac{\tilde{q}_\mu^4}{4q^2})\right)\right.$$
$$\left.\left(F_1^2-\frac{\tilde{q}_\mu^2}{4M^2}F_2^2\right)+\frac{E(p)}{2}\tilde{q}_\mu^2(F_1+F_2)^2\right]_{E_{min}(r)}^{E_F(r)}\Big\}.$$







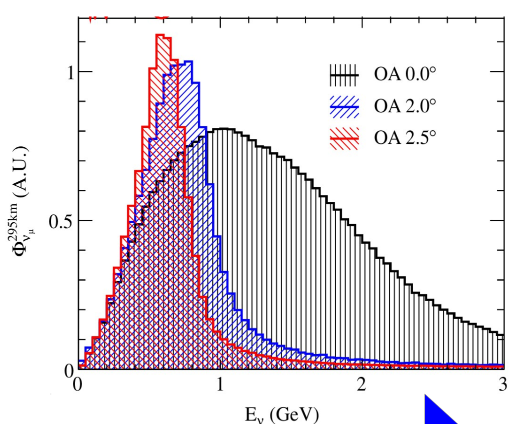
- To our Physical Review Letters result:

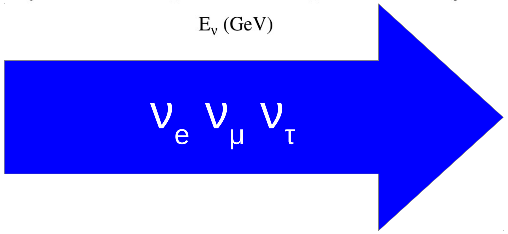
phase δ_{CP}. In this neutrino oscillation scenario, assuming $|\Delta m^2_{32}| = 2.4\times10^{-3}$ eV^2, $\sin^2\theta_{23}=0.5$, $\delta_{CP}=0$, and $\Delta m^2_{32}>0$ ($\Delta m^2_{32}<0$), a best-fit value of $\sin^2 2\theta_{13} = 0.140^{+0.038}_{-0.032}$ ($0.170^{+0.045}_{-0.037}$) is obtained.

General neutrino MC scheme

Neutrino interaction generator

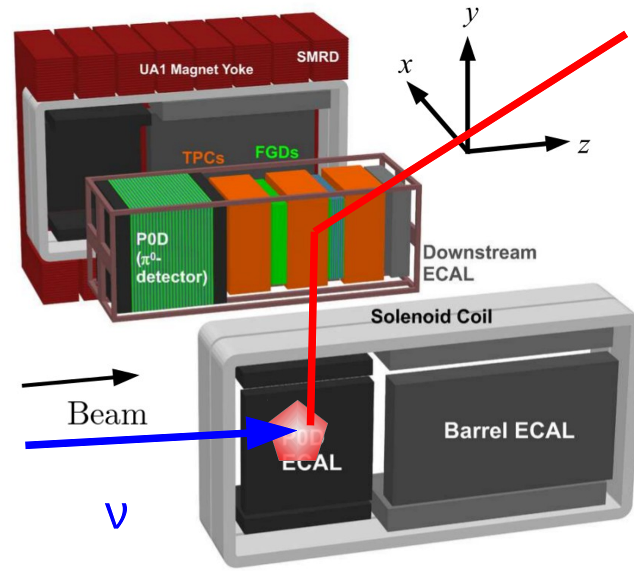
- Our focus: generators for neutrino interactions: beam profile, detector, target nucleus interaction vertex and process (dynamics), final state interactions (FSI) (e.g. GENIE, NEUT, **NuWro**)



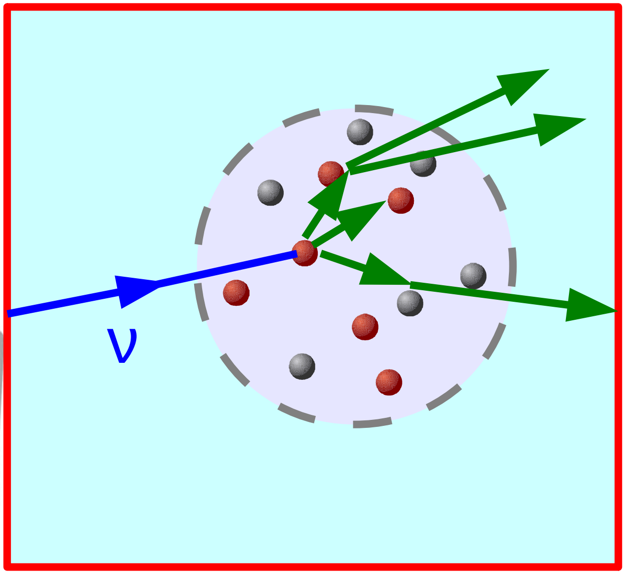


Beam profile

Detector (geometry and isotope composition)

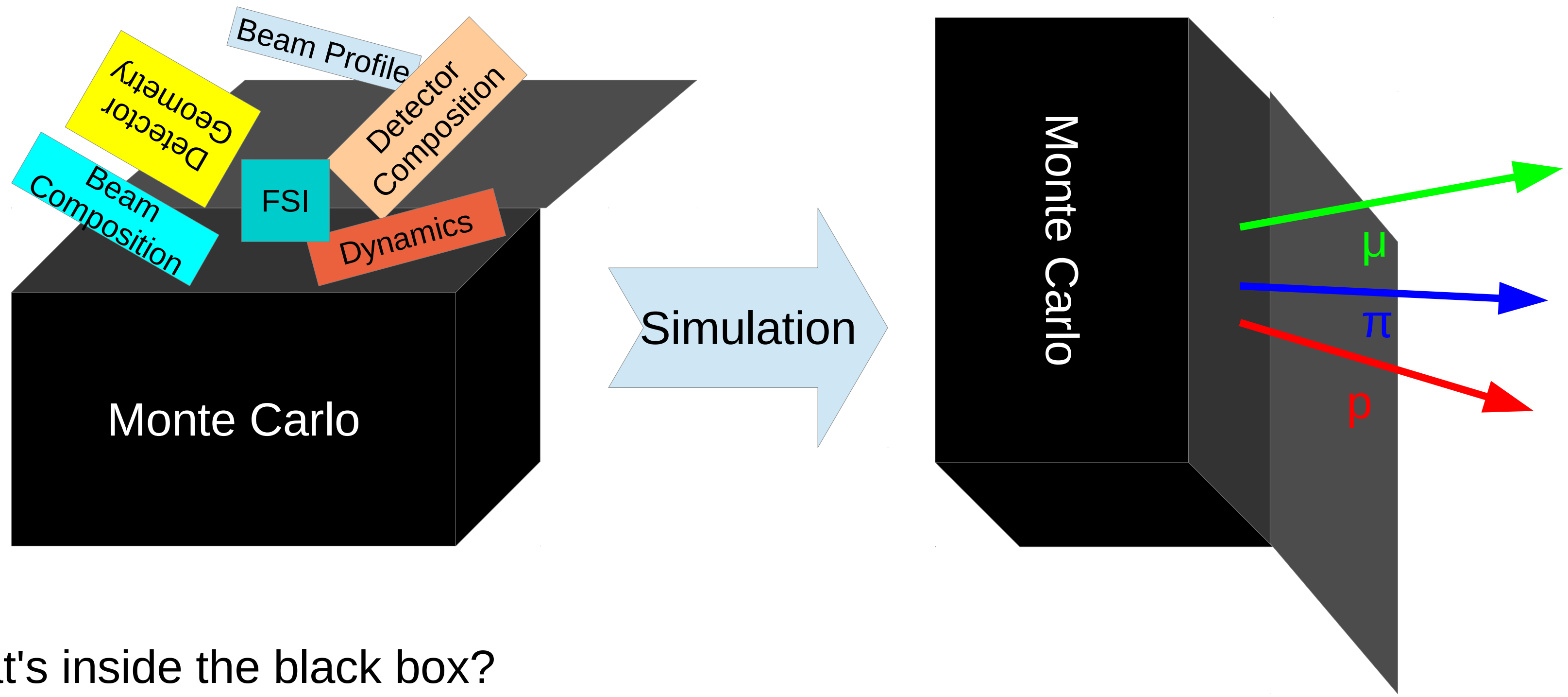


Initial and final state interactions on nuclear targets



Handling the probabilities

- Probability of drawing a neutrino flavor f with energy E interacting at point (x,y,z) with nucleus N through dynamics D producing outgoing particles $\{X_i\}$...



- What's inside the black box?

NuWro

- Main example during this talk: NuWro, the Wrocław neutrino events generator.
- The project started 2005 at the Wrocław University; an important encouragment from Danuta Kiełczewska from Warsaw
- Main authors: Tomasz Golan, Krzysztof Graczyk, Cezary Juszczak, Jarosław Nowak, Jan Sobczyk, Jakub Żmuda.
- Code written in C++ language.
- First (natural) name: Wrocław Neutrino Generator: WroNG → changed from marketing reasons... (Jan T. Sobczyk, Jaroslaw A. Nowak, Krzysztof M. Graczyk „WroNG - Wroclaw Neutrino Generator of events for single pion production” Nucl.Phys.Proc.Suppl. 139 (2005) 266)

NuWro

- **NuWro is not an official MC in any experiment and serves as a laboratory for new developments.**
- **Relatively new components (introduced or developed recently also in GENIE and NEUT):**

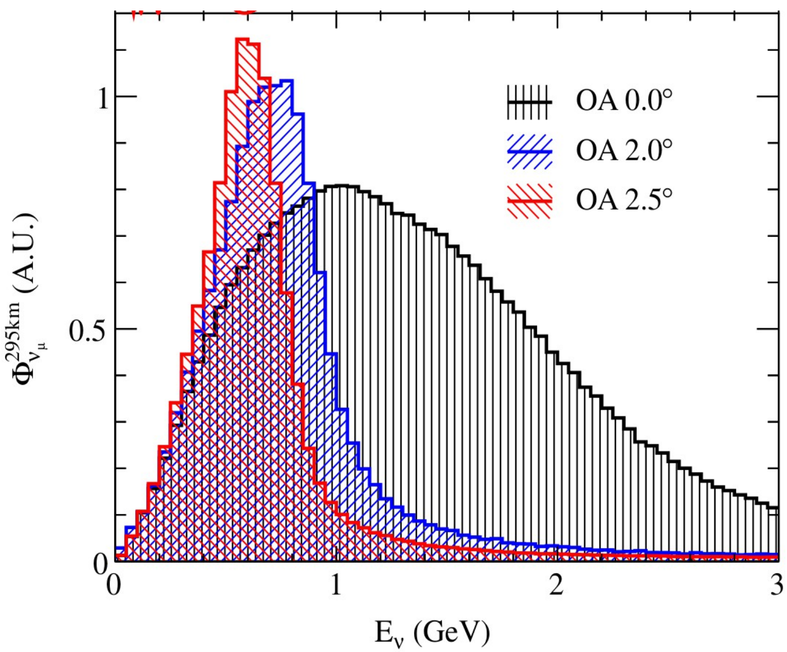
1) Meson exchange currents
2) Random phase approximation (on top of RFG)
3) Spectral function
4) **Electron simulation – coming soon!**

http://borg.ift.uni.wroc.pl/nuwro/

Repository, documentation, NuWro on-line

Beam profile

- Simple case: „perfect” beam with only one flavor:



- Uniform bin spacing, *n* bins in neutrino energy, bin width $\Delta E=(E_{max}-E_{min})/n$.
- Calculate the cummulative distribution function, invert it, or accept event according to weight~bin height?
- Actually not very effective algorithms!

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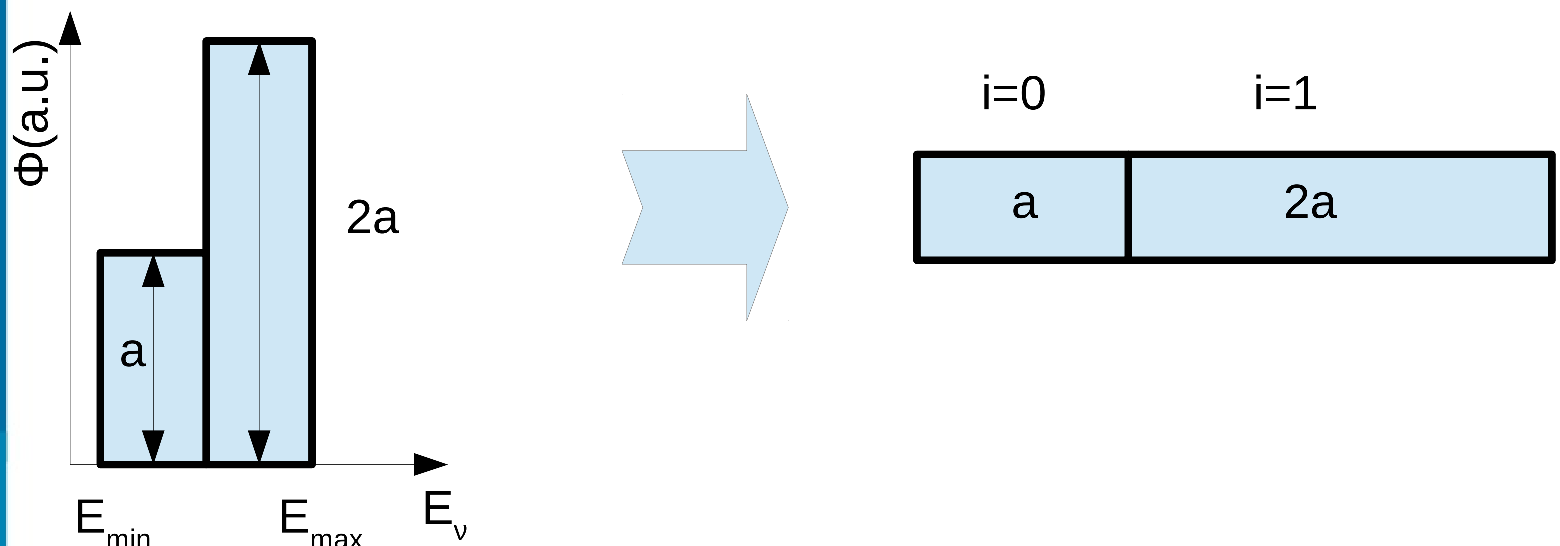
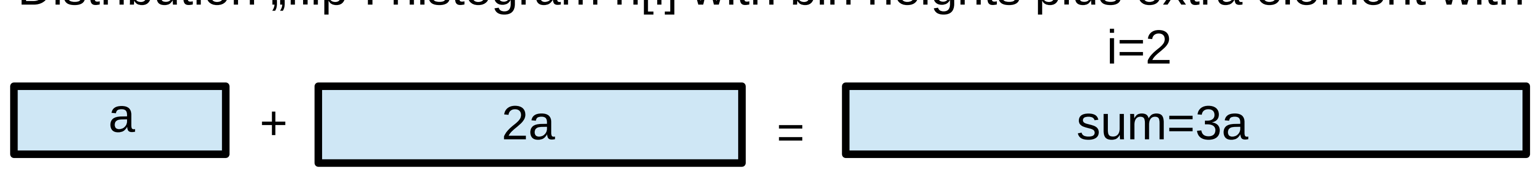
Beam profile

- Imagine „perfect” beam with only one flavor and e.g. profile given by just two bins:



- Second bin twice as probable as the first one, same widths $\Delta E=(E_{max}-E_{min})/2$.
- Distribution „flip”: histogram h[i] with bin heights plus extra element with their sum Σ



- *frand()*- random number [0,1], MT19937. ***x=frand()*sum; x<a → i=0 else i=1;***
- Uniform sampling with second bin twice as probable as first.
- After setting *i* : linear interpolation of energy inside bin (spectrum is continuous!):

$E=E_{min}+i*\Delta E + frand()*\Delta E;$

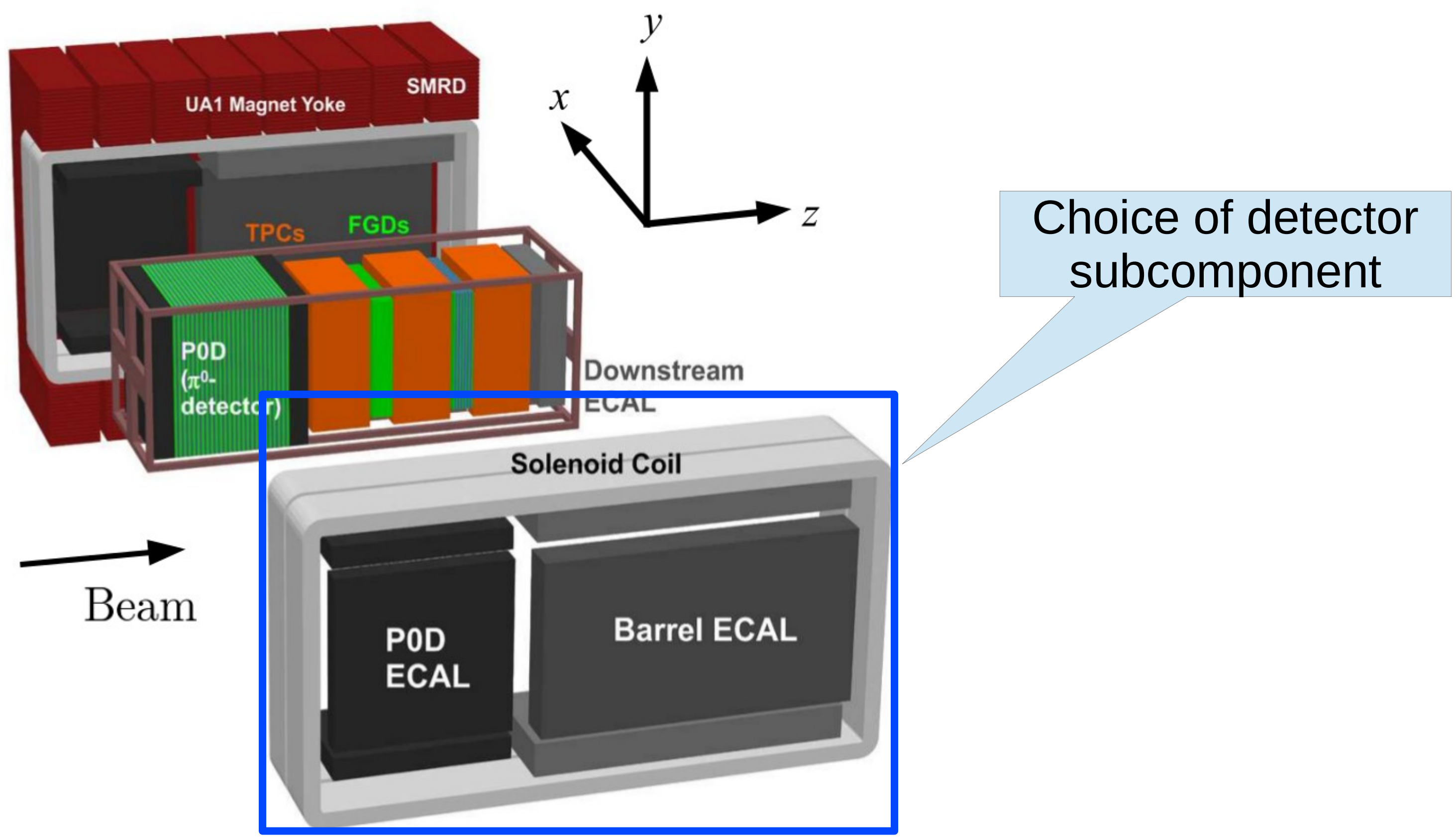
Beam profile

- Extension to any number of bins! ;)

All neutrinos with equal weights
Only some happen more often

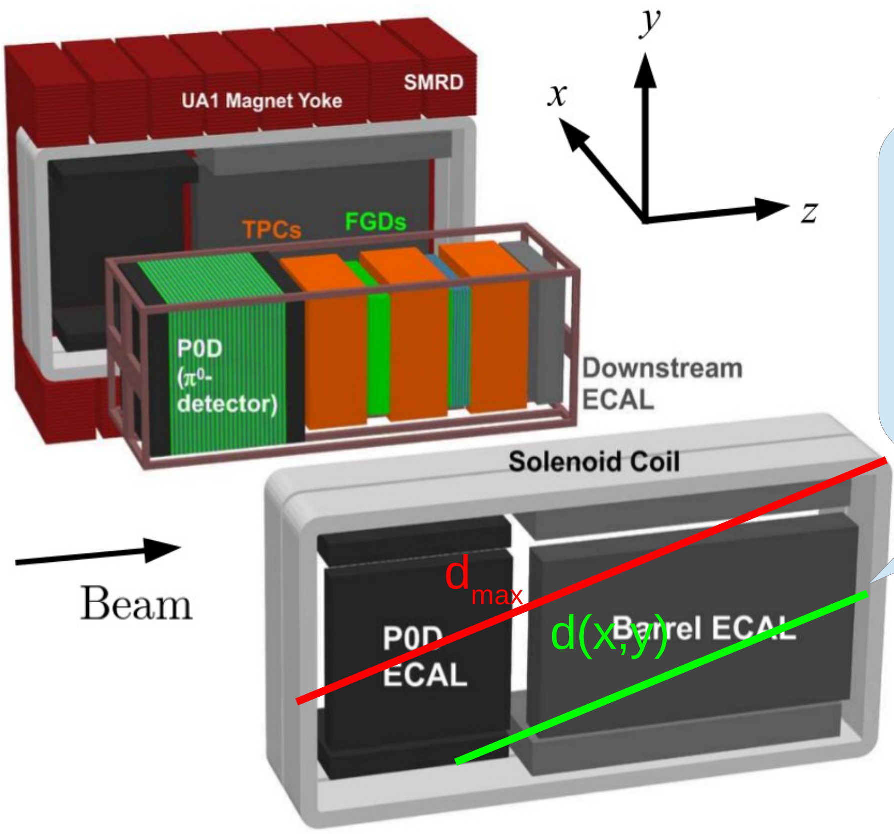
Detector

- Beam: some distribution in space+direction of neutrinos
- Detector: geometrical distribution of matter (local density and composition- fraction of isotopes).



Detector

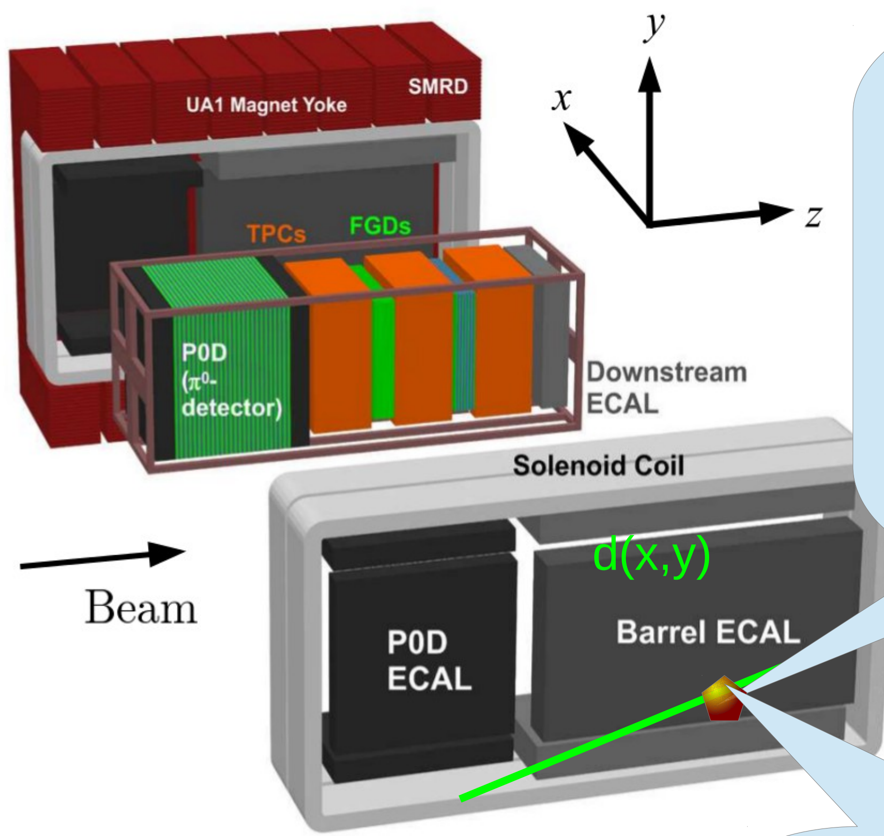
- Beam: assume neutrino direction along the z-axis.
- Detector: geometrical distribution of matter (local density and composition- fraction of isotopes).



Maximum length of neutrino trajectory inside the detector- d_{max}.
Acceptation of (x,y) with $P(x,y)=d(x,y)/d_{max}$

Detector

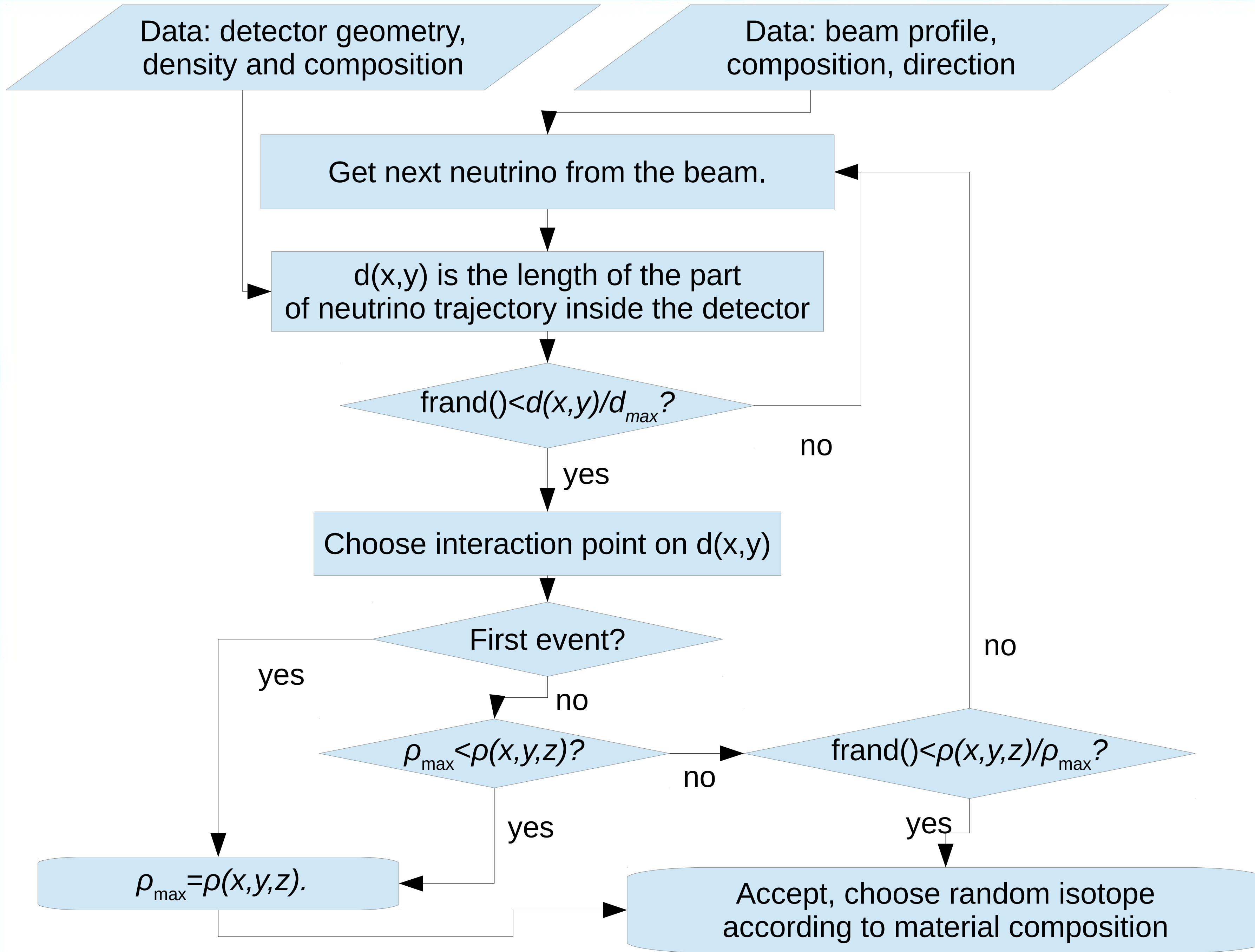
- Beam: assume neutrino direction along the z-axis.
- Detector: geometrical distribution of matter (local density and composition- fraction of isotopes).



Uniform sampling of interaction point (z) along *d(x,y)*.
First event:
$\rho_{max}=\rho(x,y,z)$.
Then acceptation with $P(x,y,z)=\rho(x,y,z)/\rho_{max}$,
(updates of ρ_{max})

Choice of target nucleus: according to local composition %Fe %O %C...

Detector


Data: detector geometry, density and composition
Data: beam profile, composition, direction
Get next neutrino from the beam.
d(x,y) is the length of the part of neutrino trajectory inside the detector
frand()<d(x,y)/dmax?
no
yes
Choose interaction point on d(x,y)
First event?
yes
no
no
ρmax<ρ(x,y,z)?
no
frand()<ρ(x,y,z)/ρmax?
yes
yes
ρmax=ρ(x,y,z).
Accept, choose random isotope according to material composition

Nuclear interaction vertex

- Two steps in each event:

A) Position of vertex inside the nucleus (density-depedent)

B) Dynamics choice: weights from flux-integrated total cross sections

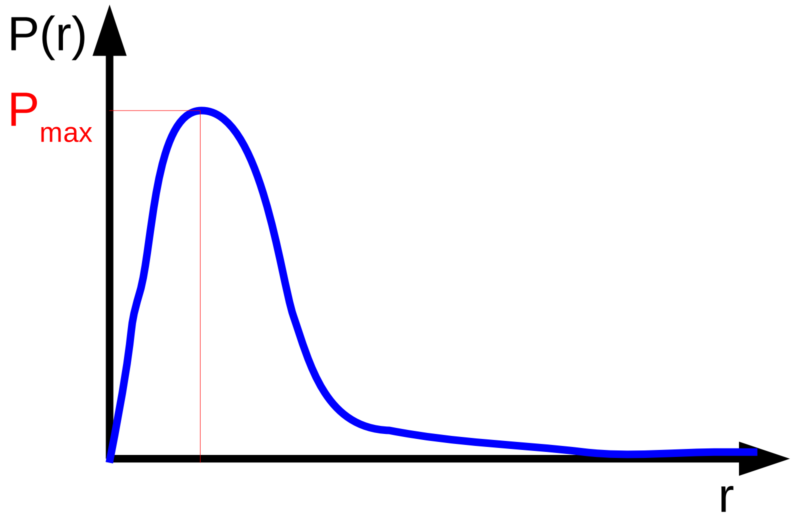
- Part A) for single-nucleon interaction: relatively easy. Each nucleus: nuclear matter density profile with spherical symmetry *ρ(r)*.
- Normalized probability:

$$P(r) = \frac{4\pi}{A} r^2 \rho(r), \int P(r)\, dr = 1$$

Nuclear interaction vertex

- To sample vertex position: find maximum probability P_{max} (efficiency/speed tip: do it only once, when your nucleus gets generated for the first time!)

$$P(r)=\frac{4\pi}{A}r^2\rho(r),\int P(r)\,dr=1$$



- Each distance → $P=P(r)/P_{max}$.
- Choose proton (P=p/(p+n)) or neutron (P=n/(p+n)). Special case: CCQE: always neutron (neutrinos) or always proton (anti-neutrinos).

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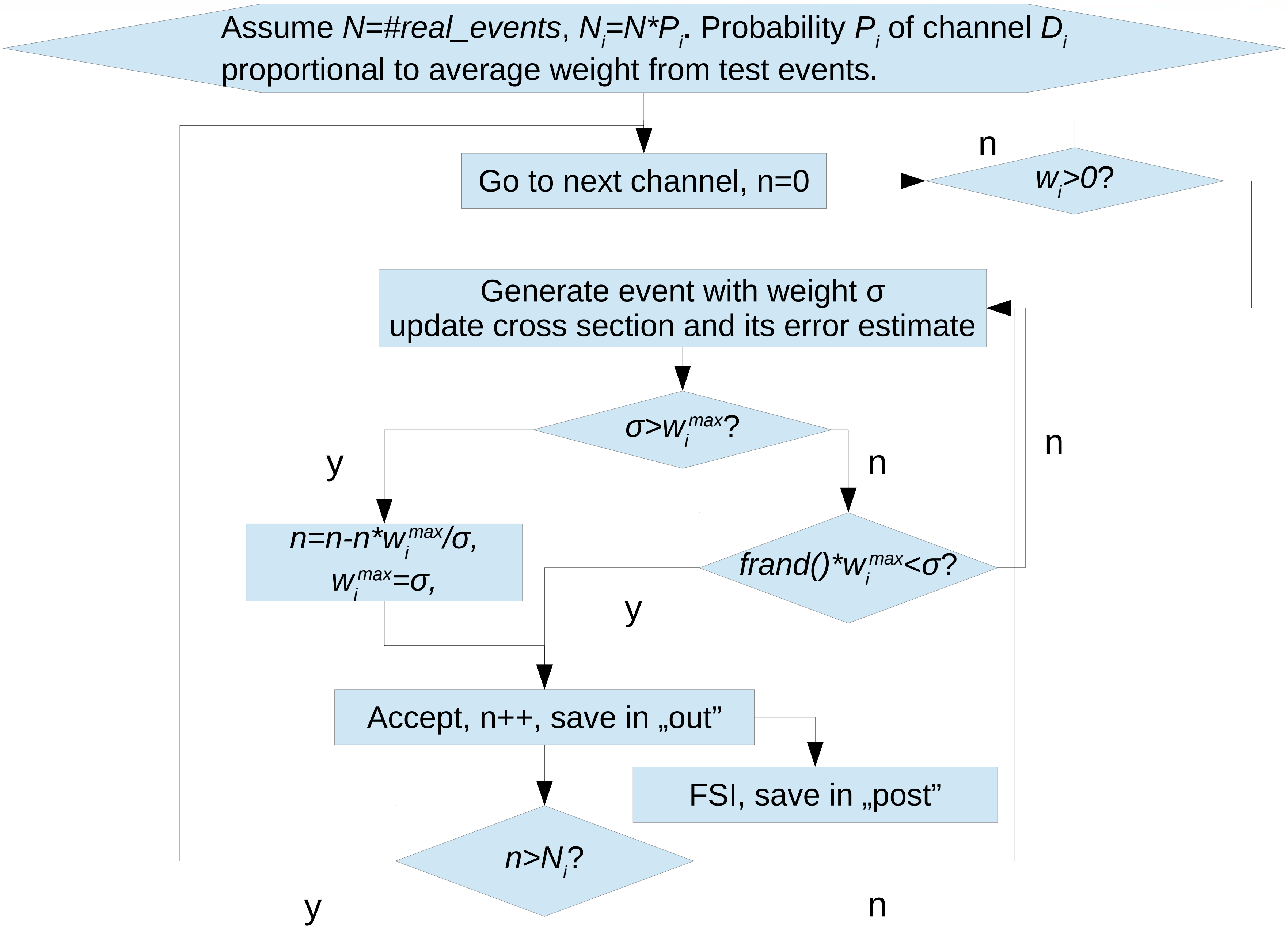
Test events

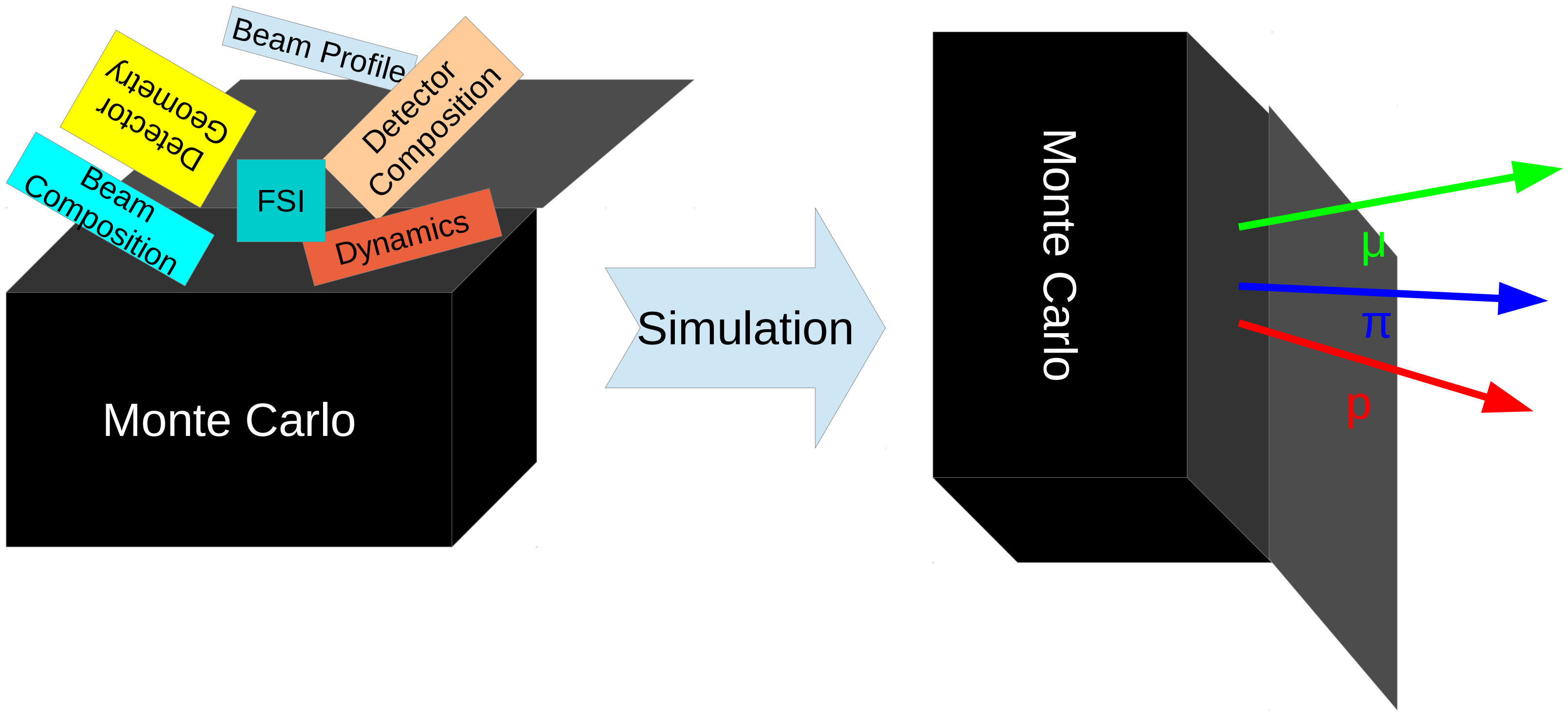
- Assume *N* channels $D_1 ... D_N$ (CCQE, NC EL, DIS, MEC...)
- No a prori knowledge of $\sigma_1 ... \sigma_N$ → „test events”. NuWro: the only generator calculating weights during run → flexibility to physical model and parameter (e.g. M_A) changes!
- For each D_i: calculation of flux-integrated total cross section- weight w_i, search for maximum differential cross section w_i^{max}.
- Test events: fast (no FSI, no save to file- unless specified otherwise!).
- Good to have as many test events, as possible, in NuWro 10 000 000 → nothing unusual.

Real events


Assume N=#real_events, N_i=N*P_i. Probability P_i of channel D_i proportional to average weight from test events.
Go to next channel, n=0
w_i>0?
n
y
Generate event with weight σ update cross section and its error estimate
σ>w_i^max?
y
n
n
n=n-n*w_i^max/σ, w_i^max=σ,
frand()*w_i^max<σ?
y
Accept, n++, save in „out”
FSI, save in „post”
n>N_i?
y
n

- Already covered:

1) General scheme of MC simulation (beam → detector → event)



- Next: general optimization „tricks”

General optimization tricks

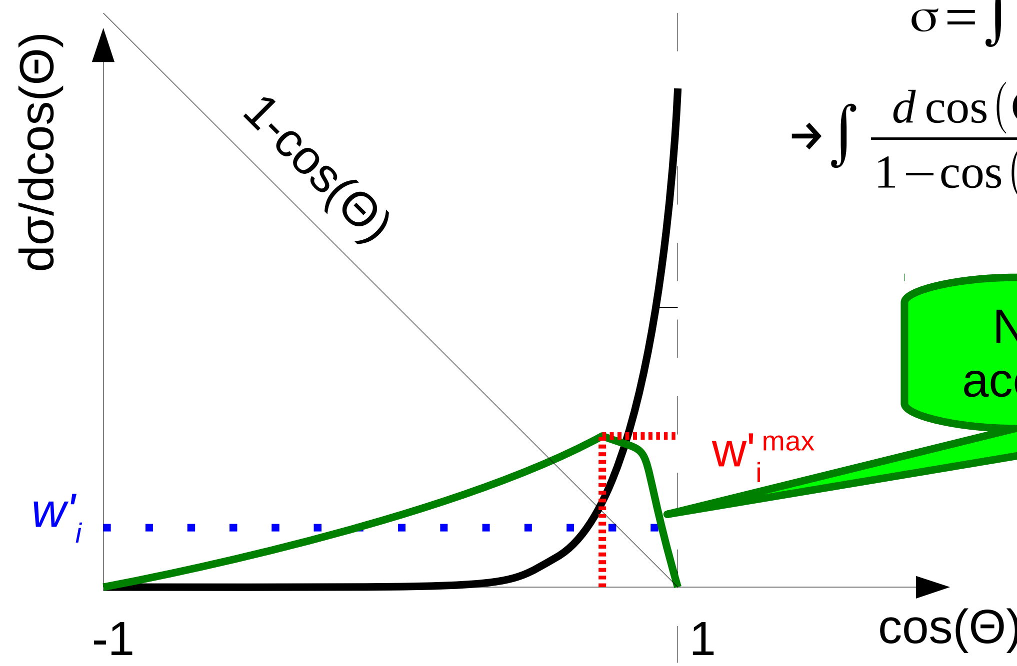
Peaked cross sections

- For QE and coherent processes: forward-peaked distributions

Typical trick: Re-weight (total XS → invariant):

$$\sigma = \int d\cos(\Theta) \frac{d\sigma}{d\cos(\Theta)} \rightarrow$$

$$\rightarrow \int \frac{d\cos(\Theta)}{1-\cos(\Theta)} \left[\frac{d\sigma}{d\cos(\Theta)} (1-\cos(\Theta)) \right]$$

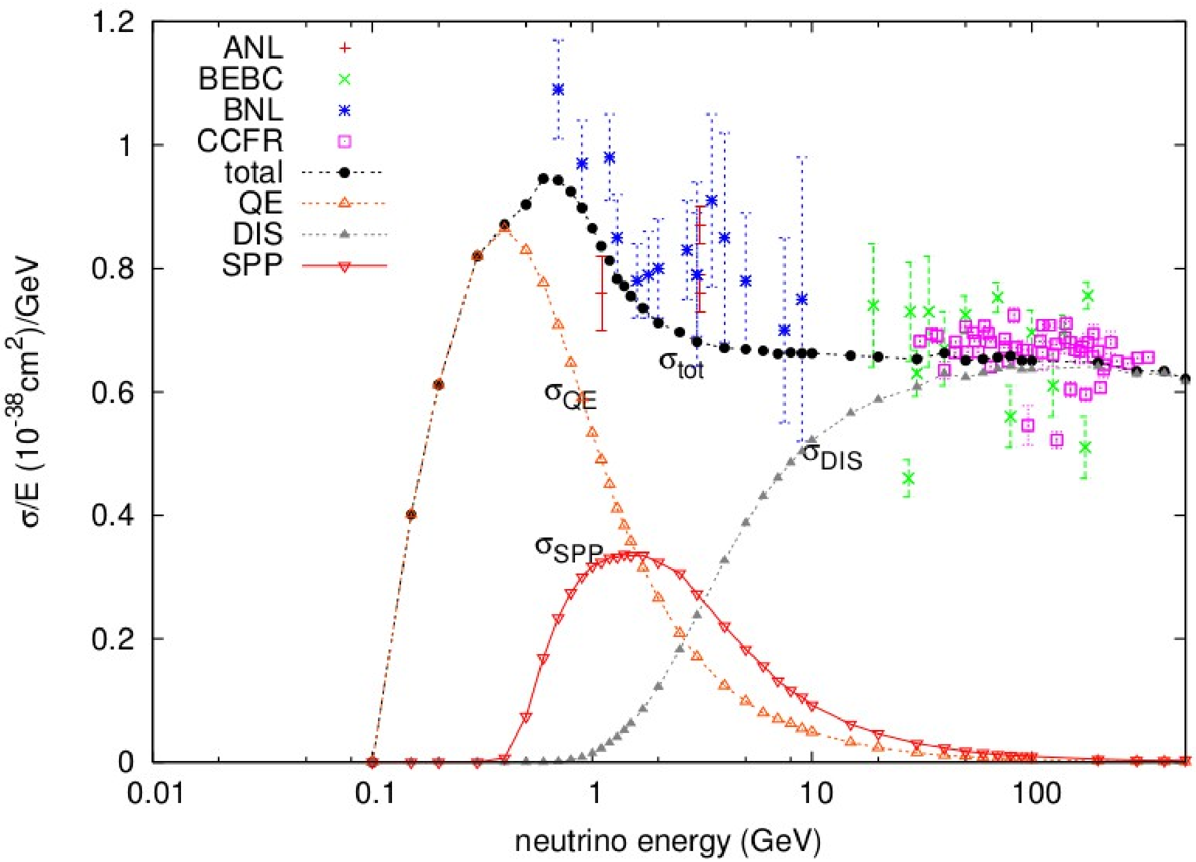


New distribution: larger acceptance and efficiency

- Acceptance according to $P=\sigma/w_i^{max}$. Very low efficiency (imagine doing 10 000 coherent events to get 2 accepted)
- Very probable large changes of w_i^{max}.

Handling physics: growing cross sections

- Another special case; Deep Inelastic Scattering
- Saturation of σ/E for isoscalar targets (source: J. Nowak PhD thesis- early NuWro):



- DIS → first rapd, than linear growth of cross section with energy.
- Small flux (above 1 GeV in T2K) and LARGE event weight (DIS cross section).

Handling physics: growing cross sections

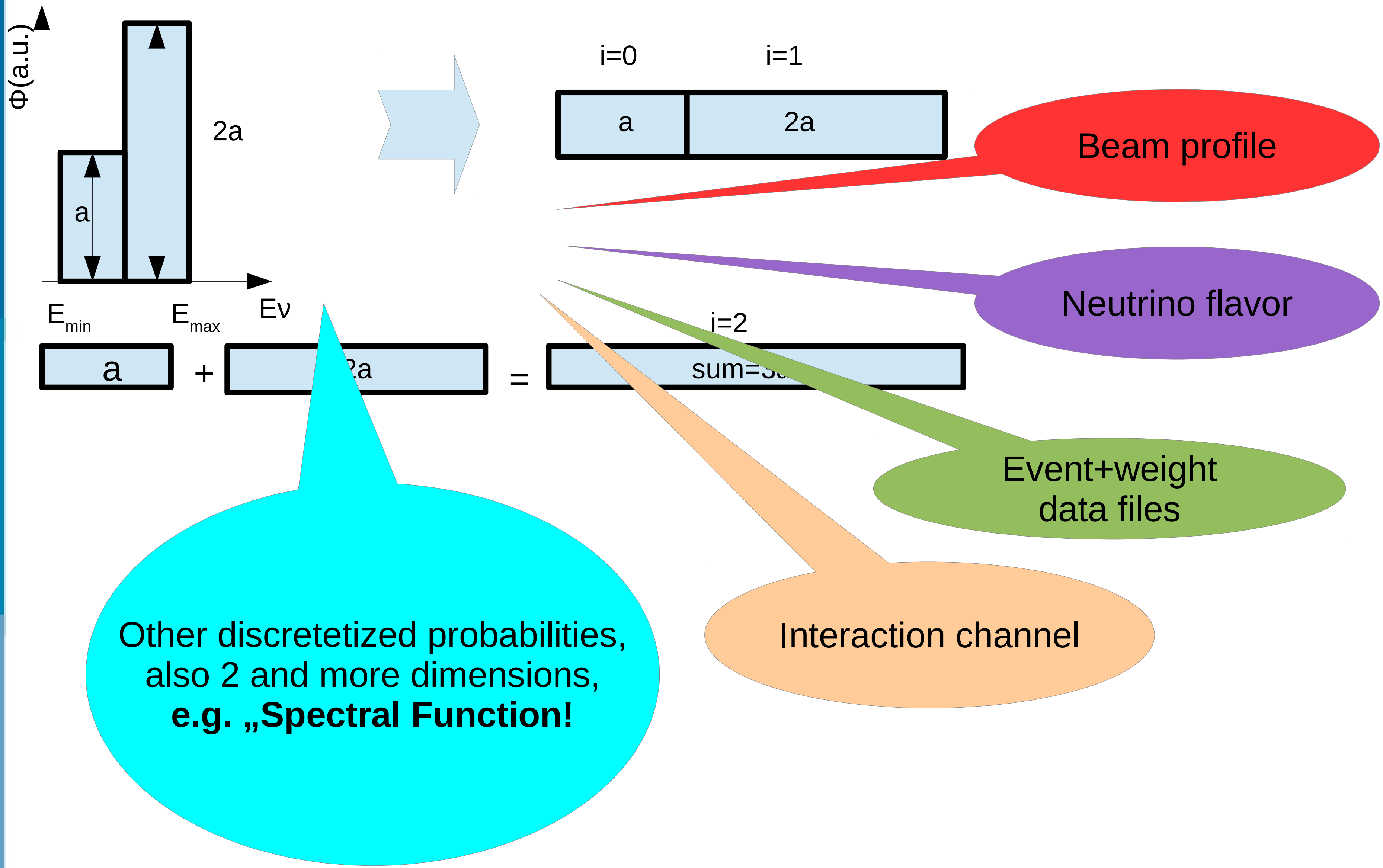
- Another (typical) re-weight:

$$P(E) \rightarrow P(E) \cdot E$$
$$\sigma(E) \rightarrow \sigma(E) \div E$$

- Fix of DIS efficiency and sampling

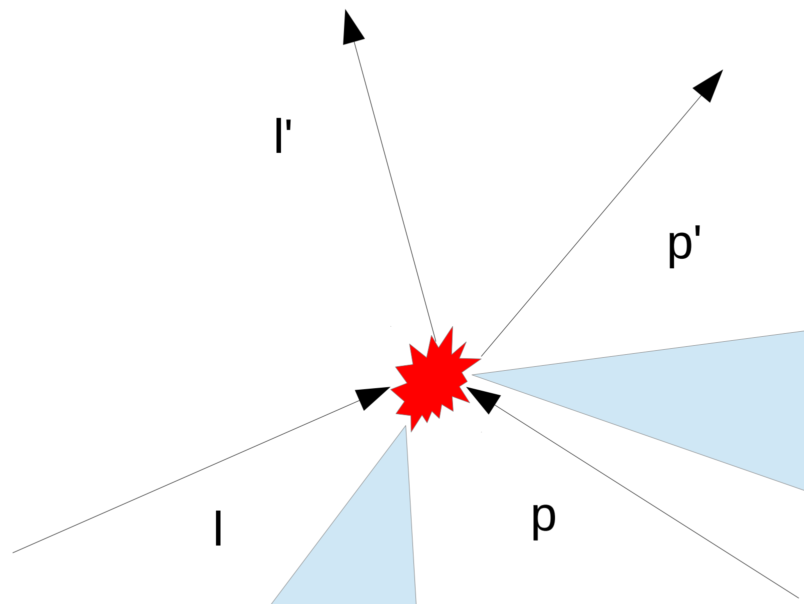
Discretized probability distributions

- „Flipped” cummulative histograms:



Frame of reference

- Typical 2+2 process (e.g. CCQE/ NCE):



Neutrino (4-momentum *l*),
Nucleon (4-momentum *p*)
→
Lepton (4-momentum *l'*, mass *m*),
Hadron (4-momentum *p*', mass *M*)

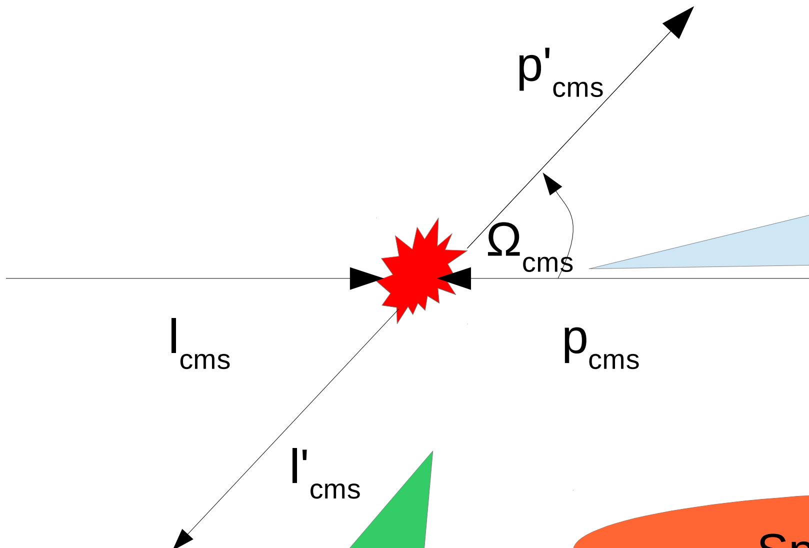
4-momentum conservation:
$(l+p)^{\mu}=(l'+p')^{\mu}$

Mandelstam variable:
$s=(l+p)^2=(l'+p')^2$

Laboratory frame= complicated phase-space
(angle-momentum dependencies in decay)

Frame of reference

- Boost to neutrino-nucleon centre-of mass frame (CMS)



4-momentum conservation:

$(l_{cms}+p_{cms})^{\mu}=(l'_{cms}+p'_{cms})^{\mu}=(s^{1/2},\mathbf{0})$

Spatial components:

$\mathbf{l}_{cms}+\mathbf{p}_{cms}=\mathbf{l'}_{cms}+\mathbf{p'}_{cms}=\mathbf{0}$

Spherical symmetry:

$$|\vec{p}\,'_{cms}|=\frac{\lambda^{1/2}(s,m^2,M^2)}{2\sqrt{s}}$$

$$\lambda(x,y,z)=(x+y+z)^2-4(xy+yz+zx)$$

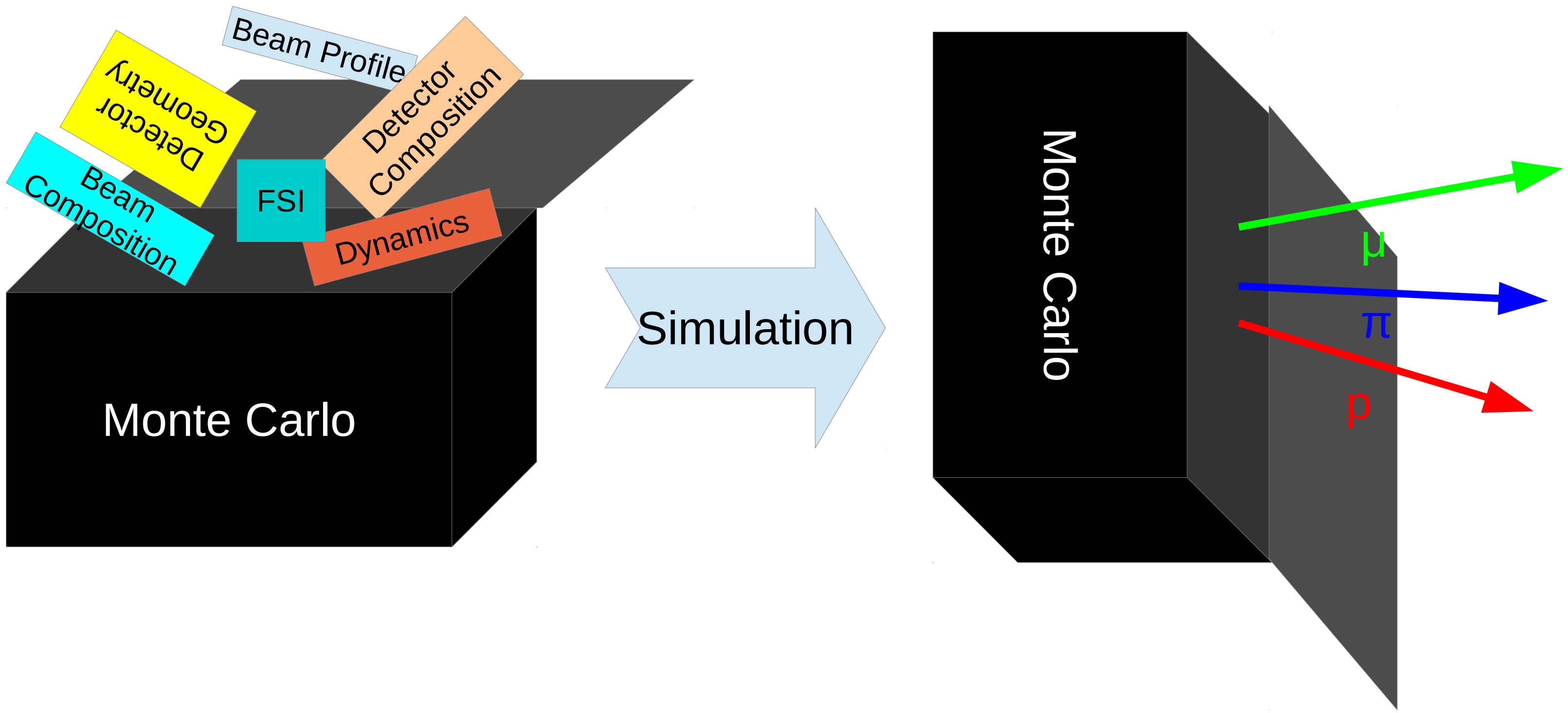
Easy phase-space , easy limits, e.g.

$$Q^2_{min/max}=-m^2_{l'}+\frac{E_{\nu\,cms}}{\sqrt{s}}(s+m^2_{l'}-M^2\pm\lambda^{1/2}(s,M^2,m^2_{l'}))$$

- Already covered:

1) General scheme of MC simulation (beam → detector → event)

2) General optimization tricks (peaked or growing cross sections, sampling from discretized distributions, frame of reference choice)

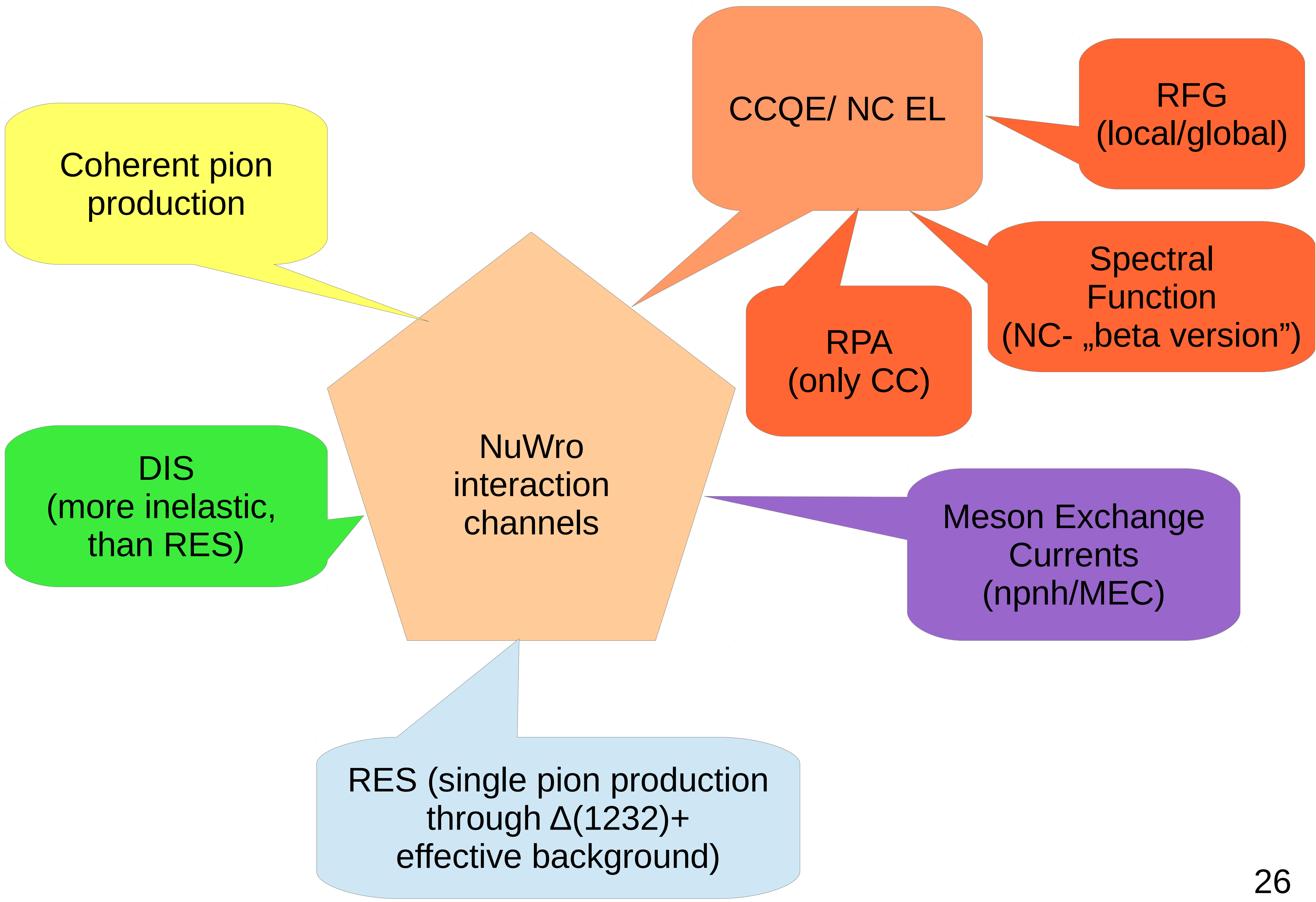


- Next: physical models

Part II

Selected interaction channels

Physical channels in NuWro



Physical channels in NuWro

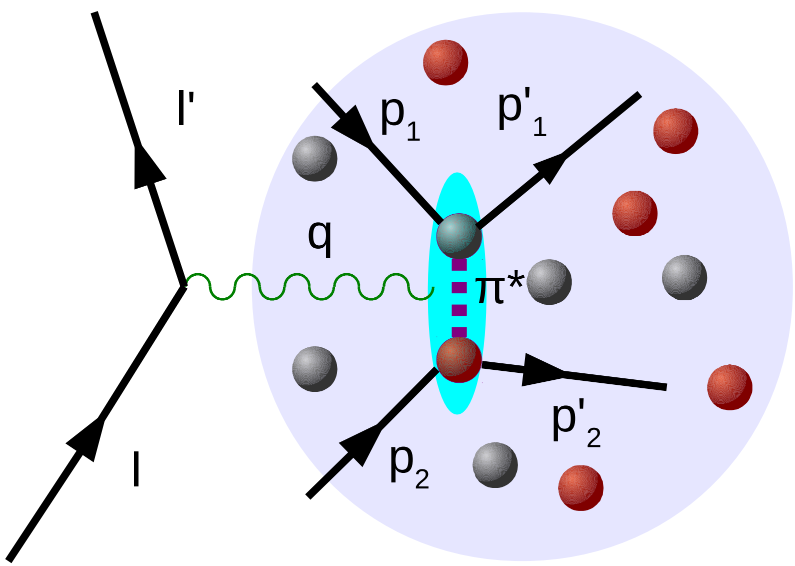
- **<u>Relatively new components (introduced recently also in GENIE and NEUT):</u>**

1) Meson exchange currents
2) Random phase approximation (on top of the RFG)
3) Spectral function

Topic of this section

Meson Exchange Currents

- MEC: growing interest in neutrino community
- First proposals of MEC search in neutrino interactions in T2K!
- MEC „cartoon”:



- Need for MC implementation.
- In NuWro three models: Marteu-Martini-like, Transverse Enhancement and **Valencia** model.
- Each theoretical model above → **inclusive muon double-differential cross sections, no information about nucleon kinematics**

MEC Hadronic model

- Information about actual nucleon dynamics: unavailable → effective ansatz.
- Microscopic models predicting inclusive cross sections: (local) Fermi gas ground state → two (or three) random nucleons from local denstiy distribution (NuWro).
- Problem: around 20% nucleons in strongly correlated proton-neutron pairs with back-to-back momenta → developing version with correlated nucleons with momenta randomized from spectral function (J. Sobczyk's talk in Seattle)

MEC Hadronic model

- Vertex position inside the nucleus:

1) Two nucleons at the same point in space, probability ~ρ^2.

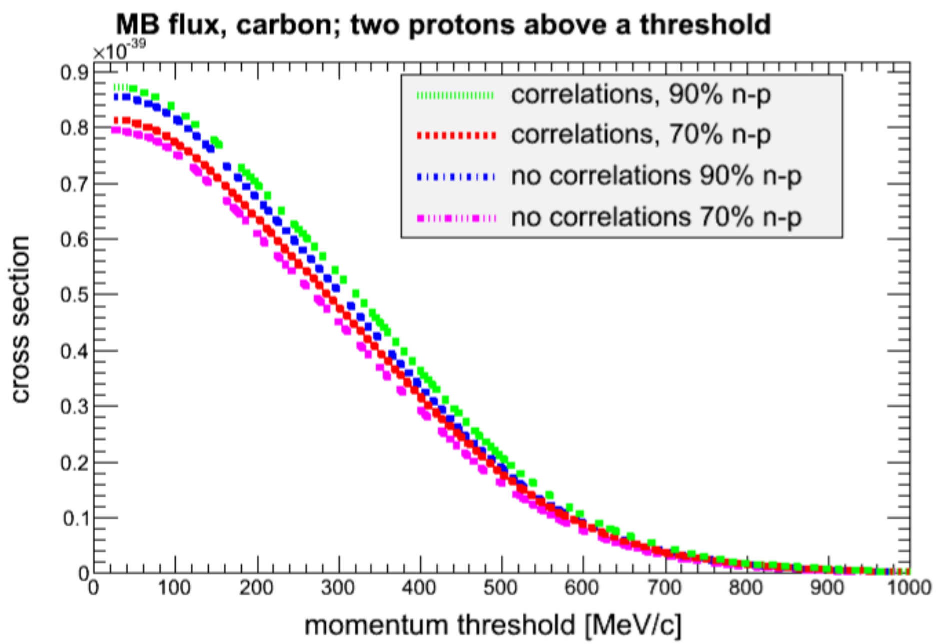
2) Two nucleosns at different points in space: both from single-particle distribution ~ρ.

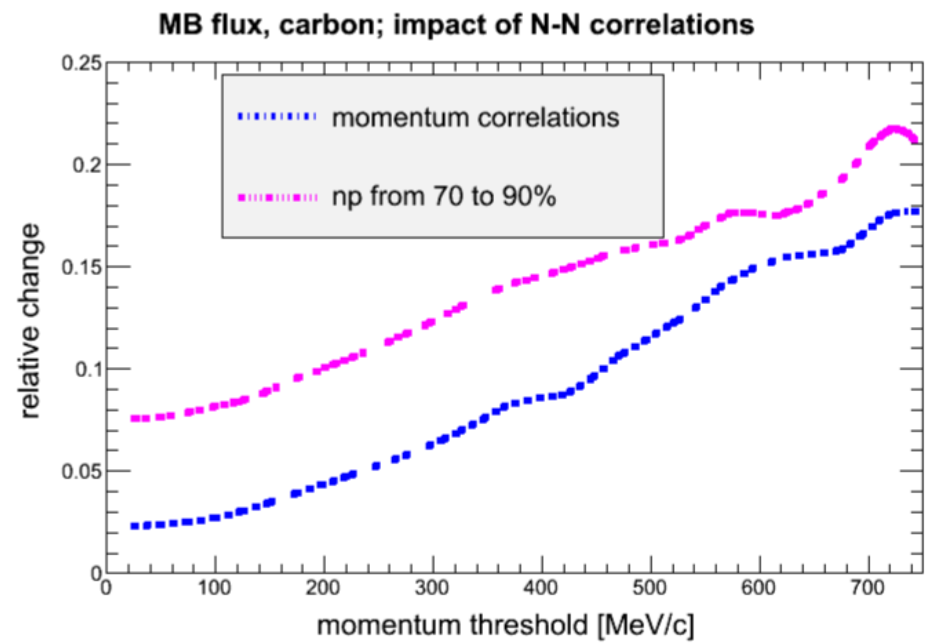
- Second solution: different (local) Fermi momenta, used for Valencia implementation.
- Isospin content: in NuWro free parameter (default 60% mixed p-n initial pairs)

MEC Hadronic model

From J. Sobczyk's talk in Seattle

Impact of correlation effects on number of proton pairs in the final state:

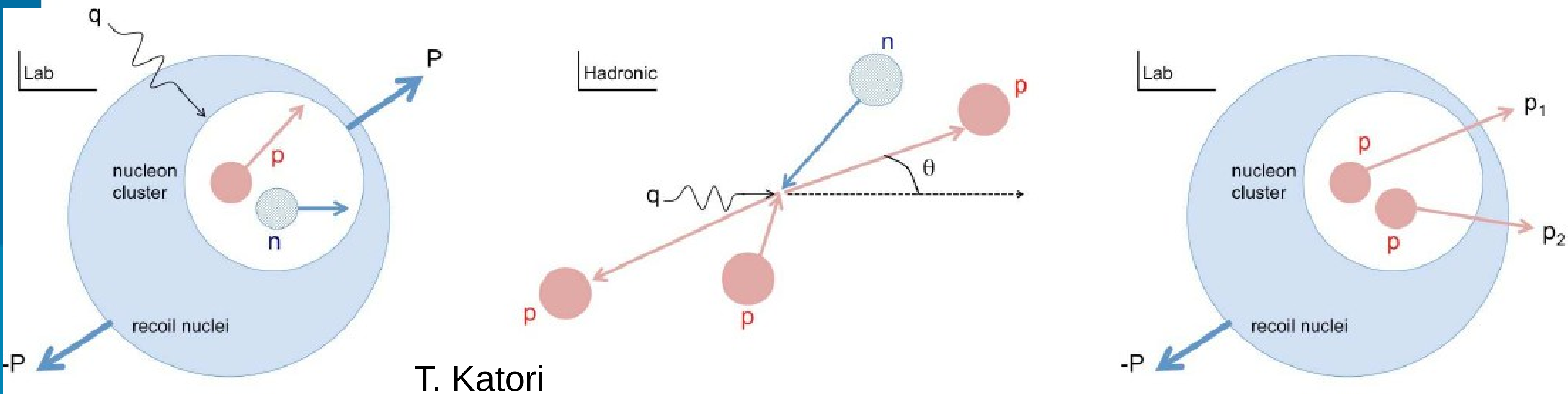




Isospin and momentum correlations are analyzed seperately. A possible confusion: In above figures correlations means initial state nucleon momenta are back-to-back.

MEC Hadronic model

- algorithm by J. Sobczyk Phys. Rev. C86 015504 using hadronic CMS:



T. Katori

The same in each MEC muon inclusive cross section model:

Lepton kinematics/ 4-momentum in laboratory frame → Inclusive cross section (weight) → Random nucleons from Fermi Sea → Hadronic CMS: isotropic decay → Laboratory frame, optional Pauli blocking

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MEC cross section

- Example of Valencia MEC model: even with numerical approximations (J. Nieves, I. Ruiz-Simo, M.J. Vicente-Vacas Phys.Rev. C83 (2011) 045501) 5-fold integrals inside double-differential cross section (main model prediction):

$$W^{\mu\nu}_{\rm 2p2h}(q) = \Theta(q^0)\frac{1}{M^2}\int\frac{d^3r}{2\pi}\sum_{N,N',\lambda}\int\frac{d^4k_\pi}{(2\pi)^4}\Theta(q^0-k^0_\pi)F^2_\pi(k_\pi)\mathrm{Im}\overline{U}_R(q-k_\pi,k_F^N,k_F^{N'})A^{\nu\mu}\times$$

$$\times D^2_\pi(k_\pi)F^2_\pi(k_\pi)\frac{f^2_{\pi NN}}{m^2_\pi}\vec{k}^2_\pi\Theta(k^0_\pi)\mathrm{Im}U_\lambda(k_\pi)$$

„Exact” theory, but no time for that in MC simulation!

Need for an ***effective*** MC implementation, highly nontrivial:

Either cross section tables or „response functions” (now in NuWro)

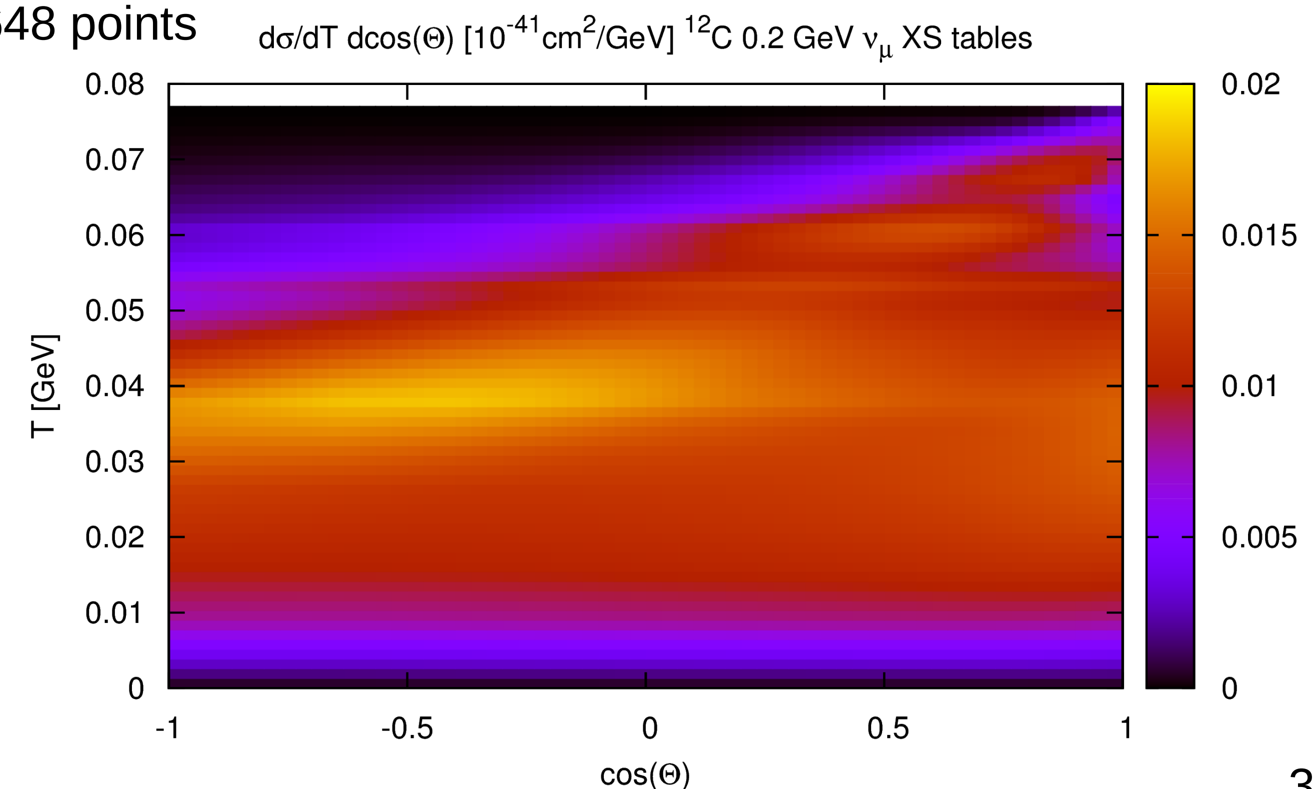
1) Accuracy of MC

2) Code versatility

3) Code speed

MEC cross section

- Usual approach: discrete tables for chosen kinematic variables (e.g. E_ν, T_μ, $\cos(\Theta_\mu)$ → first attempt in NuWro, then NEUT).
- Limited energy range problem (first:series from J. Nieves only ~3 GeV, then extension up to 30 GeV)
- Optimal binning dependent on flavor, antineutrinos etc. → usually non-uniform.
- Linear interpolation for each nucleus 92 (E) x 31 (cos(Θ)) x 31 (T) x 2 (flavors) x 2 (antineutrinos)=353648 points

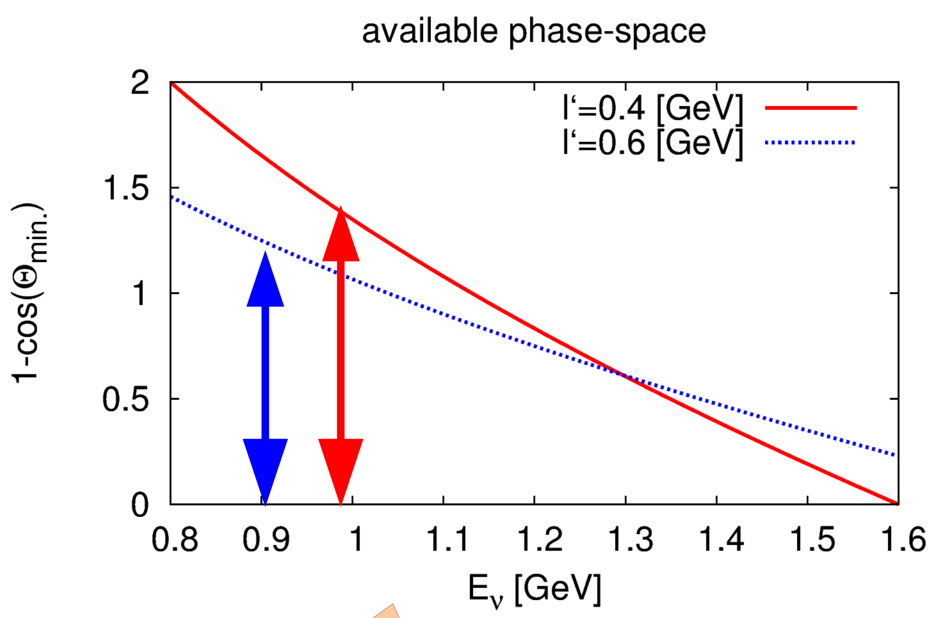


MEC cross section

- Higher energies: cut in momentum transfer to q_{max}=1.2 GeV. Above: effective field theory failure, R. Gran, J. Nieves, F. Sanchez and M.J. Vicente Vacas Phys.Rev. D88 (2013) 113007.

Rapid phase-space collapse

$$\cos(\Theta)_{min} = \frac{E_\nu^2 + \vec{l}\,'^2 - q_{max}^2}{2 E_\nu |\vec{l}\,'|}$$



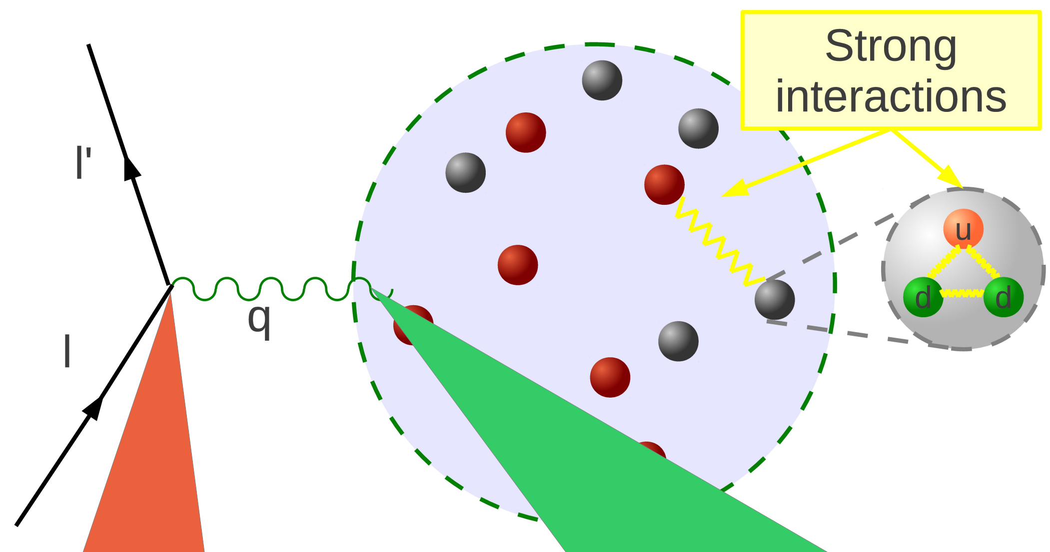
Very large number of tables in neutrino energy (92)

Existence of momentum transfer cut: disadvantage or advantage?

Different interpolation for high and low energies

MEC cross section

- So far separate tables: energy, target, flavor, antiparticle...
- Point of view of nucleus (one boson exchange (OBE), no polarization):



Nucleus **does not „know”:**

1) Lepton mass (flavour)

2) Lepton energy

Nucleus **„knows”:**

1) How hard you hit it (energy and momentum transfer)

2) Interaction type (neutrino/antineutrino CC/NC or charged lepton electromagnetic without information on particle/antiparticle)

- **Nucleus „responds” only to what it „knows”!**

MEC cross section

- Unpolarized inclusive double-differential neutrino cross section:

$$\frac{d^3\sigma}{d\Omega' dE'} = \frac{|\vec{k}'|E' M_i G^2}{\pi^2}\Big\{2W_1(q^0,|\boldsymbol{q}|)\sin^2\frac{\Theta'}{2} + W_2(q^0,|\boldsymbol{q}|)\cos^2\frac{\Theta'}{2}$$

$$- W_3(q^0,|\boldsymbol{q}|)\frac{E+E'}{M_i}\sin^2\frac{\Theta'}{2} + \frac{m_l^2}{E'(E'+|\vec{k}'|)}\Big[W_1(q^0,|\boldsymbol{q}|)\cos\Theta' +$$

$$- \frac{W_2(q^0,|\boldsymbol{q}|)}{2}\cos\Theta' + \frac{W_3(q^0,|\boldsymbol{q}|)}{2}\left(\frac{E'+|\vec{k}'|}{M_i} - \frac{E+E'}{M_i}\cos\Theta'\right)$$

$$+ \frac{W_4(q^0,|\boldsymbol{q}|)}{2}\left(\frac{m_l^2}{M_i^2}\cos\Theta' + \frac{2E'(E'+|\vec{k}'|)}{M_i^2}\sin^2\Theta'\right) +$$

$$- W_5(q^0,|\boldsymbol{q}|)\frac{E'+|\vec{k}'|)}{2M_i}\Big]\Big\}$$

5 nuclear „response functions”

Dependence on nucleus type, channel, energy and momentum transfer **only!** Antineutrino: W_3 sign change

Due to the m_l^2/E^2 dependence of W_4, W_5 → only W_1, W_2, W_3 really matter

Knowledge of Wi = knowledge of double-differential cross sections for each flavor, antineutrinos and with no neutrino energy limits

MEC cross section

- Idea: keep all complicated cross sections as structure functions:

1) No need for separate tables in neutrino energy → no upper limit.
2) No need for separate tables for flavors.
3) No need for separate tables for antineutrinos.
4) Same binning always.
5) Because 4) → simple algorithm for all cases, e.g. linear interpolation with uniform step → gain in speed.
6) Smaller data set (Carbon+Oxygen muon/electron (anti) neutrino=353 648 points, response function grid Carbon+Oxygen 2*5*120*121/2 = 72 600 points).
7) „Natural” cut in momentum transfer.

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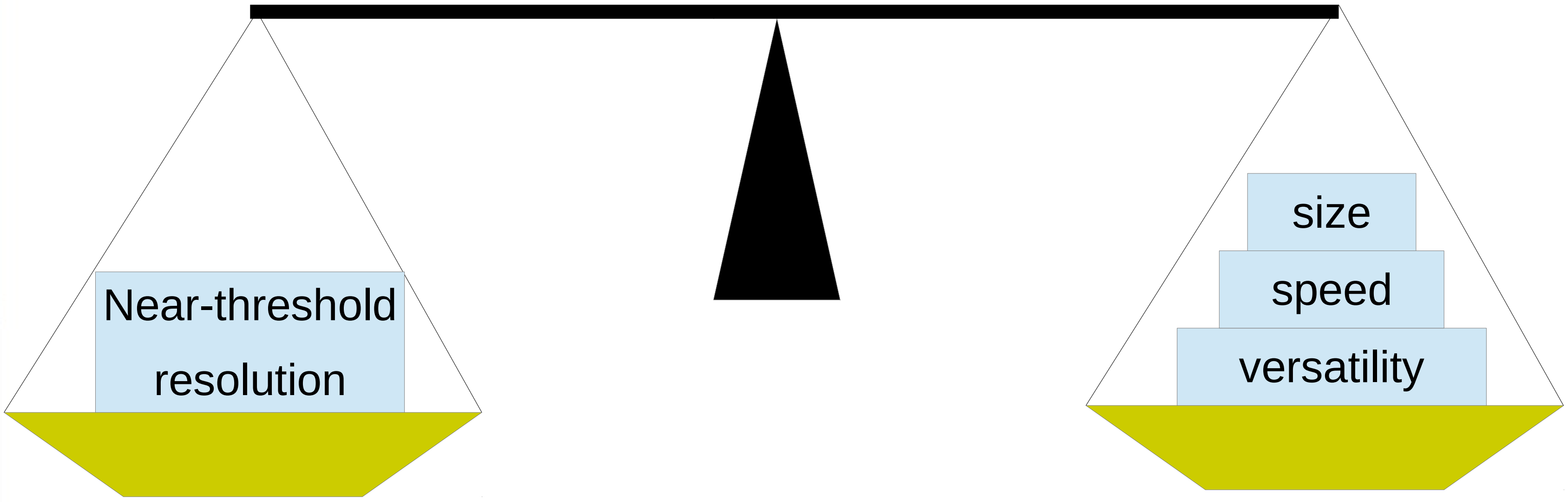
MEC cross section

- Valencia MEC: limited region ($q^0<|\boldsymbol{q}|$ and limited $|\boldsymbol{q}|$). Other models: response saturation hypothesis, extrapolation for higher values.
- Warning: grid step in q^0= grid step in T_μ. E.g. 10 MeV step in q^0 for 200 MeV muon neutrino = 8 available points in T_μ for interpolation in kinetic energy → possible resolution loss near MEC threshold, but at T2K peak ~600 MeV almost 50 points!
- Near threshold: small beam intensity and small MEC cross section, not a real problem?
- For $E\text{-}m_l>q_{cut}$ saturation of resolution (whole grid available).

MEC cross section

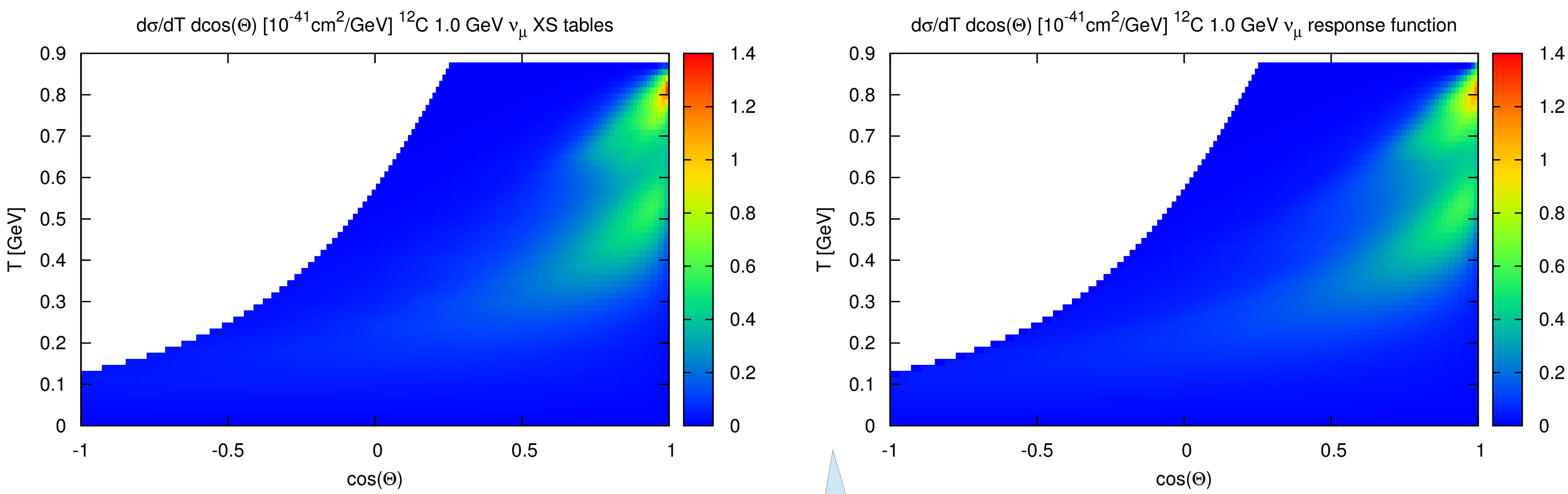
- Thanks to courtesy of J. Nieves and M. J. Vicente Vacas: code for MEC hadronic tensor element production = code for structure functions.
- 10x10 MeV grids for Carbon, Oxygen and Calcium up to momentum cut (NuWro).
- Only physical region stored ($q^0<|\mathbf{q}|$).
- Our dilemma:



MEC cross section

- Sample double-differential cross sections for 1 GeV ν_μ scattering off ^{12}C.

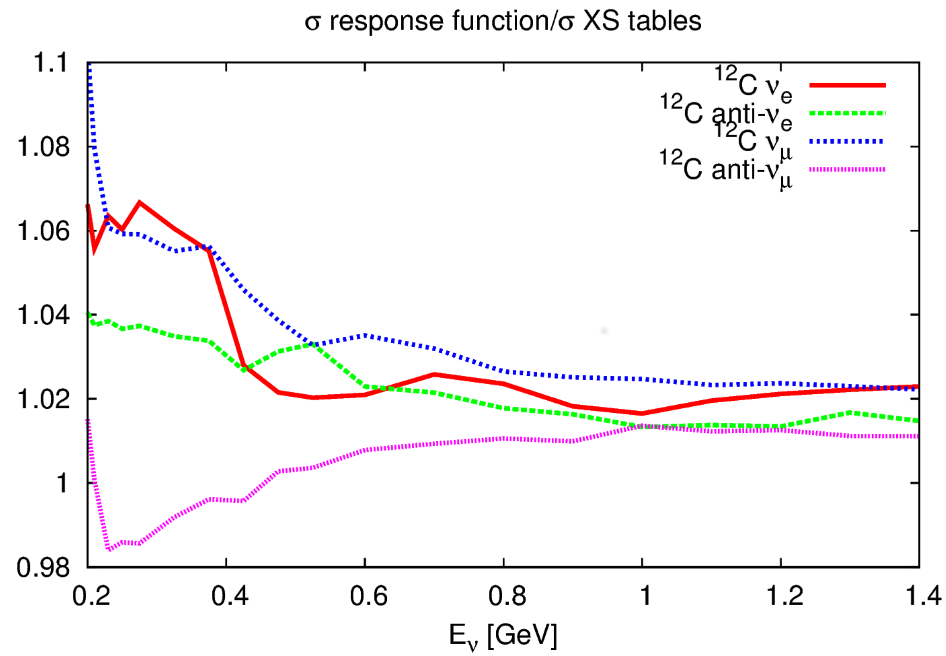
(left- from cross section tables, right – from response functions)



Seemingly identical!

MEC cross section

- Relative difference in total cross sections:



- At 0.2 GeV difference up to 10%, above 0.5 GeV: differences below 4%, 1-2% at 1 GeV.
- Near 0.2 GeV: small cross section, small T2K flux, at the verge of detector possibilities → no problem, posiible more dense binning
- Response function approach valid!
- For theoretical models predicting inclusive cross sections: store response functions not cross sections.

MEC cross section

- Whenever possible, do analytic kinematic limits. e.g. Valencia MEC model solutions both for energy transfer and scattering angle:

$$\cos(\Theta)_{min}=\frac{E_\nu^2+\vec{l}\,'^2-q_{max}^2}{2E_\nu|\vec{l}\,'|}<1\rightarrow E_\nu^2+(E_\nu-q^0)^2-m_l^2-q_{max}^2-2E_\nu\sqrt{(E_\nu-q^0)^2-m_l^2}<0$$

- For each randomized neutrino energy → limits, then:

1) Evaluate phase space in energy transfer and in scattering angle.
2) Sample inside allowed phase-space.
3) Calculate cross section (event weight).

- Less zero weight events, bigger efficiency

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Spectral Function

- Spectral Function: replacement of usual (local) Fermi distribution for quasielastic event by a probability distribution of removing nucleon with momentum ***p*** leaving the residual nucleus with excitation energy *E*. Extra integral in cross section:

Residual nucleus

Initial nucleus, E_0

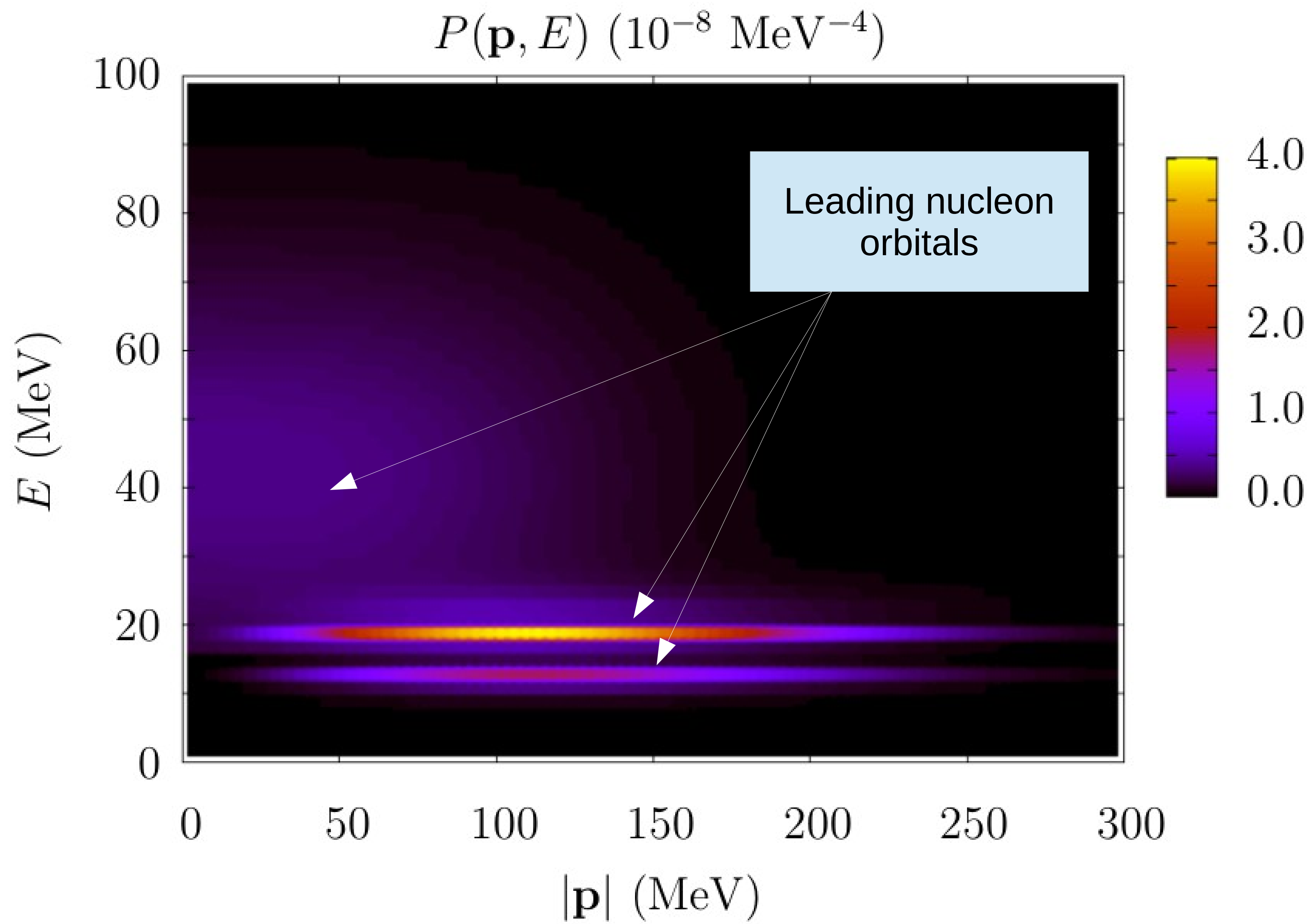
$$\Theta(k_F - |\vec{p}|) \to \int P(\vec{p}, E)\, dE = \int \sum_n |\langle \Psi_n^{A-1} | a_{\vec{p}} | \Psi_0^A \rangle|^2 \delta(E_0 + E - E_n)\, dE$$

Nucleon removal (annihilation operator)

- Mix of theoretical mean-field calculation (shell model orbitals) and short-range correlations with experimental data on actual orbital occupation numbers and momentum spreadings plus a lot of phenomenological „cooking”.
- Works of Omar Benhar's group
- In NuWro: implementation based on A. Ankowski PhD thesis by C. Juszczak

Spectral Function

- Example Oxygen SF:

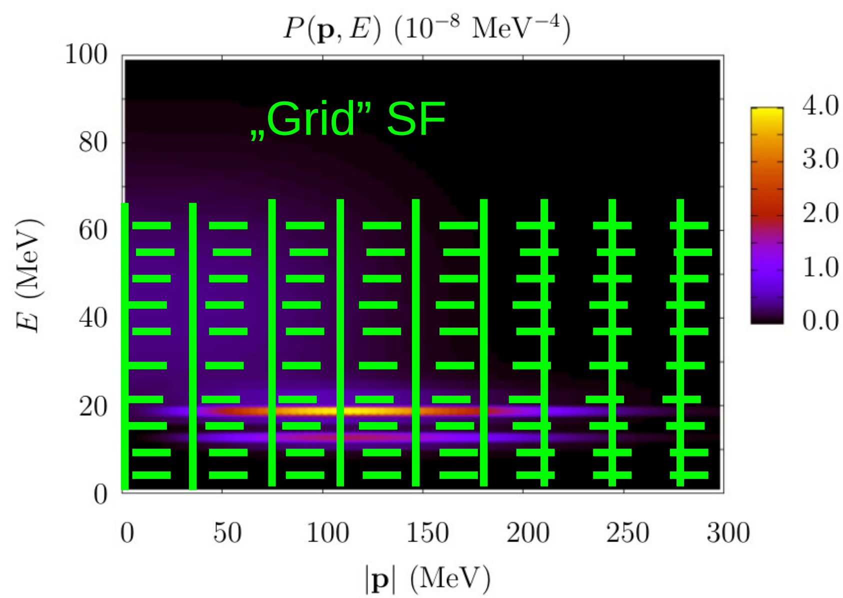


Spectral Function

- Again, *P(**p**,E)* first-principle computation too complicated for MC.
- Response function: good for cross sections, but here- nucleon kinematics!
- Storage of *P(**p**,E)* (two methods in NuWro):

1) Two-dimensional grid in momentum and removal energy „grid SF”

2) Effective SF with removal energy probabilities as a vector of gaussians [central value E_{0i}, width w_i, norm N_i] → A. Ankowski PhD thesis



$$P(E) = \sum_i N_i \exp\left(-\frac{(E - E_{0i})^2}{2w_i^2}\right)$$

- Grid SF: ^{12}C, ^{16}O, ^{40}Ar, ^{56}Fe, gaussian SF ^{16}O, ^{40}Ca, ^{40}Ar.
- **No lepton FSI, which change differential cross section shapes (electron/muon energy re-distribution -different FSI from hadronic ones)!**

Spectral Function

- Both cases:

1) Create spectral function (find a way to do it once for the first interaction with given target!), get *P(p)* → integration (gaussian) or sum (grid) of *P(p,E)* w.r.t. *E*. (possible pre-calculation and storage in data files → some CPU time saved).
2) Get neutrino from the beam.
3) Get interaction point from LFG density distribution.
4) Sample momentum *p* according to *P(p),* uniform sample direction.
5) Sample *P(E|p)*.
6) Boost to neutrino-nucleon CMS.
7) Uniform decay.
8) Boost back to laboratory frame
9) Check Pauli Blocking

Typical for QE (steps 2–3)

SF only! (steps 4–5)

Typical for QE, save for PB (steps 6–9)

Spectral Function

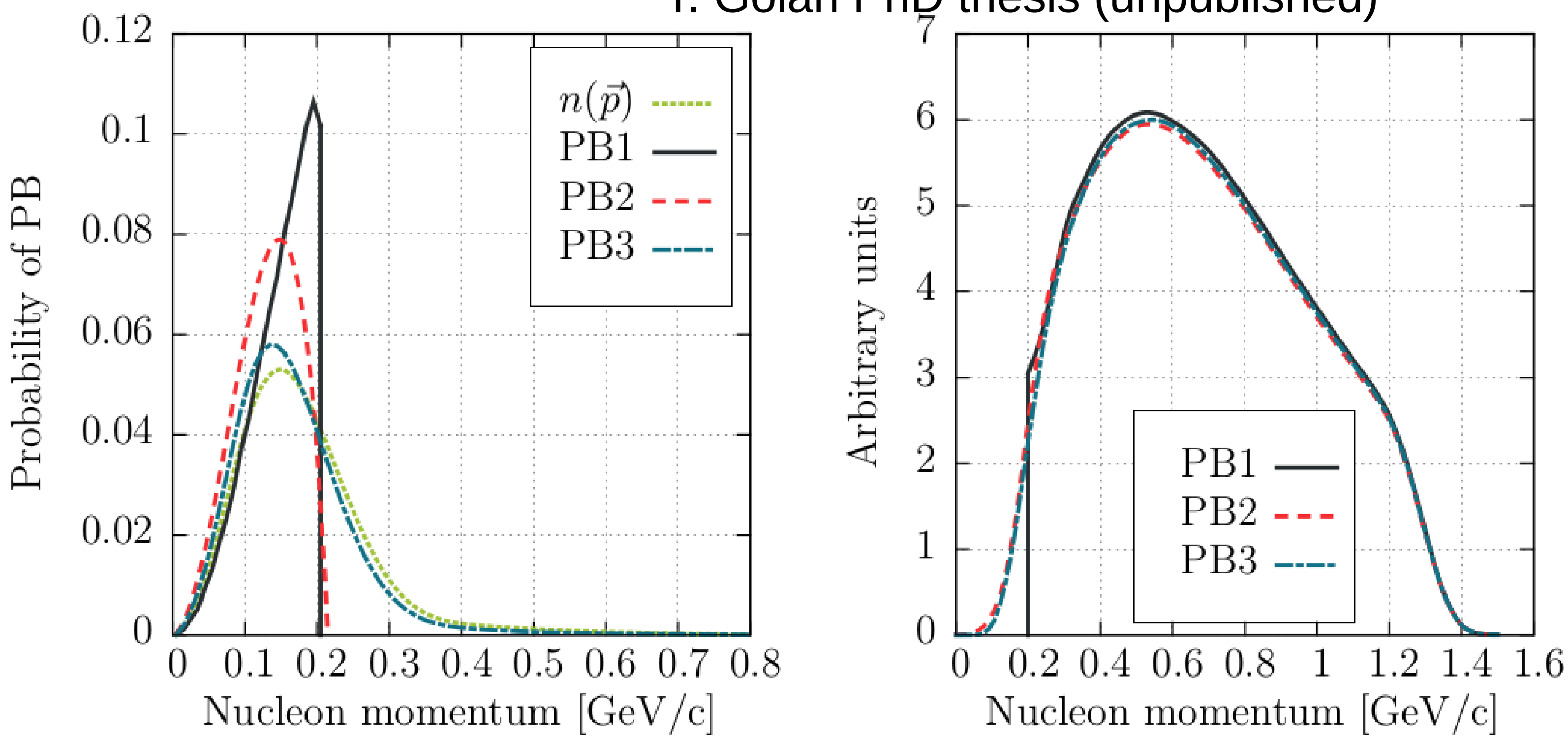
- Pauli Blocking in SF MC: not exactly obvious:

1) First method: mean Fermi momentum → sharp cutoff

2) Second method: interaction point from local density distribution → local Femi momentum → better, smooth distribution (NuWro)

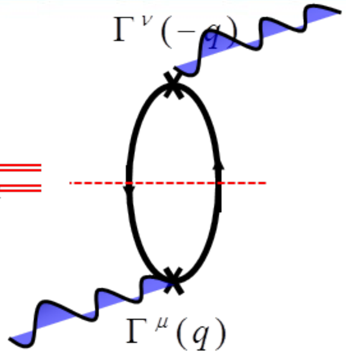
3) Third method: probability $P(p_f)$ translated for occupational number for final momentum state $n(p_f)$: check frand() against $n(p_f)$ → closest to actual SF physics, not standard in MC.

T. Golan PhD thesis (unpublished)



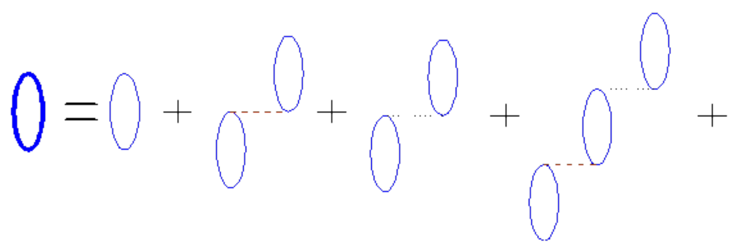
Random Phase (Ring) Approximation

$\mathrm{Im}\ \Pi^{\mu\nu}_{RFG} =$



K. Graczyk

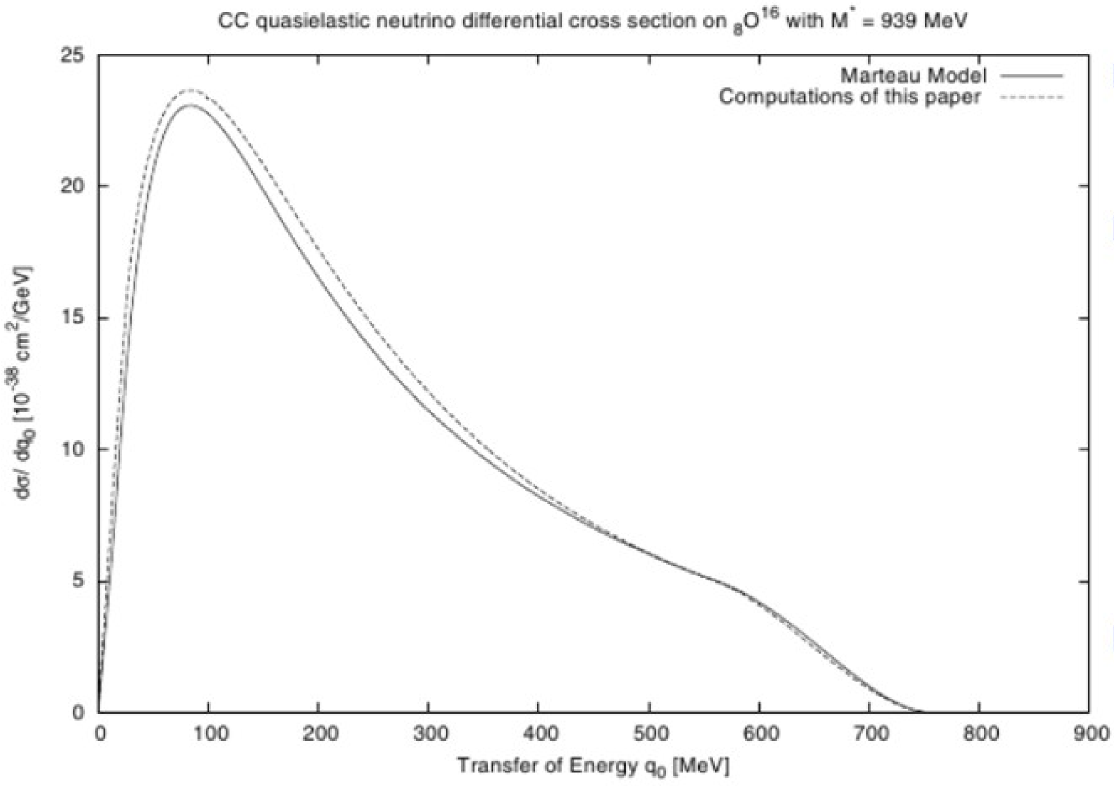
$$i\Pi^{\mu\nu}(q) = \int \frac{d^4p}{(2\pi)^4} Tr\left(G(p+q)\Gamma^\mu G(p)\Gamma^\nu\right)$$



- Algebraic solution of Dyson equation (by K. Graczyk – relativistic Ring Approximation)

Random Phase (Ring) Approximation

NuWro RPA implementation



- a part of K.M. Graczyk PhD thesis
- based on a theoretical paper K.M. Graczyk, JTS, *The algebraic solution of RPA for the CC quasielastic neutrino nucleus scattering*, Eur. Phys. J C31 (2003) 177
- implementation (C. Juszczak): a CCQE event is generated; its weight is multiplied by

K.M. Graczyk, JTS, Eur. Phys. J C31 (2003) 177

For QE Marteau is basically the same as Martini model.

K. Graczyk

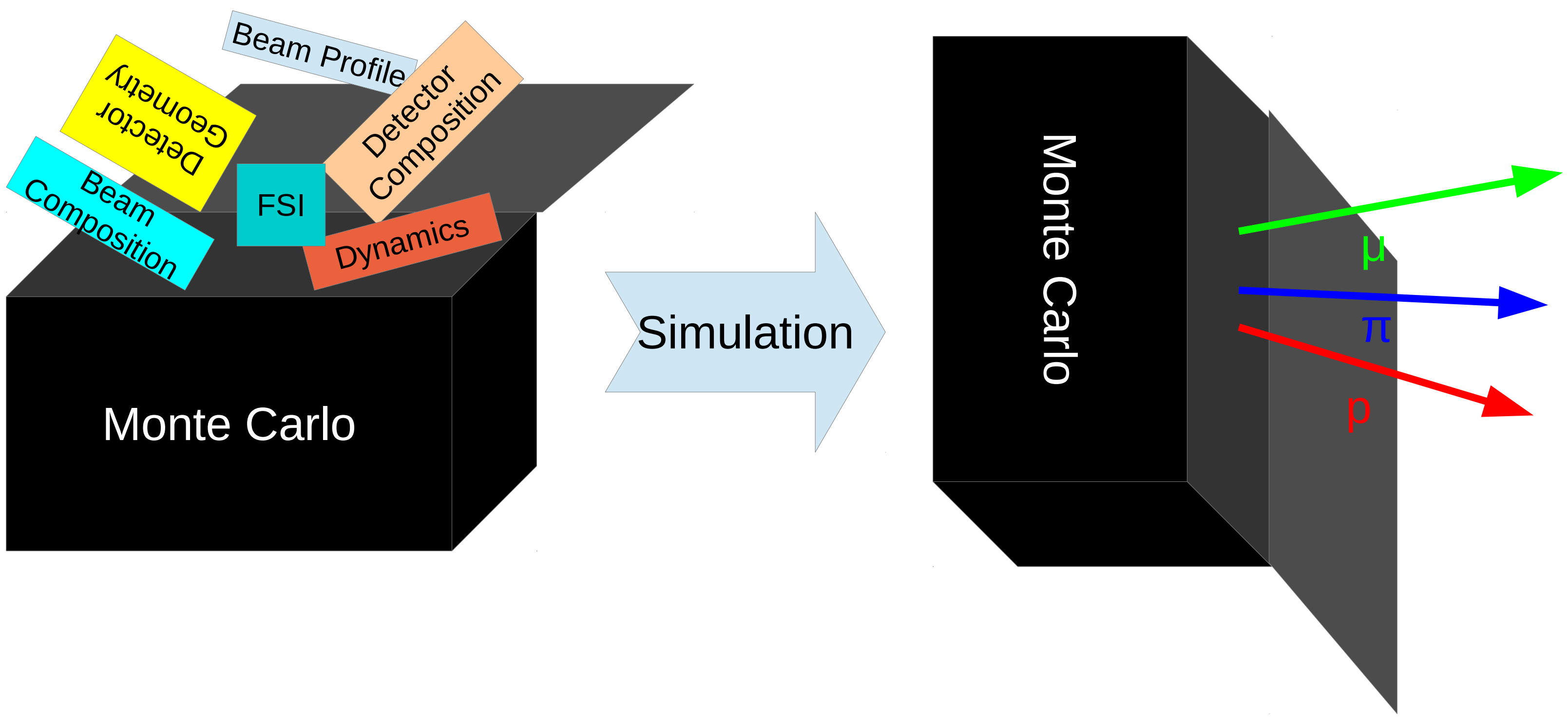
$$\frac{\dfrac{d^2\sigma^{RPA}}{d^2T_\mu d\cos\theta}}{\dfrac{d\sigma^{LFG}}{dT_\mu d\cos\theta}}$$

- Already covered:

1) General scheme of MC simulation (beam → detector → event)

2) General optimization tricks (peaked or growing cross sections, sampling from discretized distributions, quick sampling from Fermi ball, frame of reference choice)

3) Handling complicated interaction models (MEC/SF/RPA)

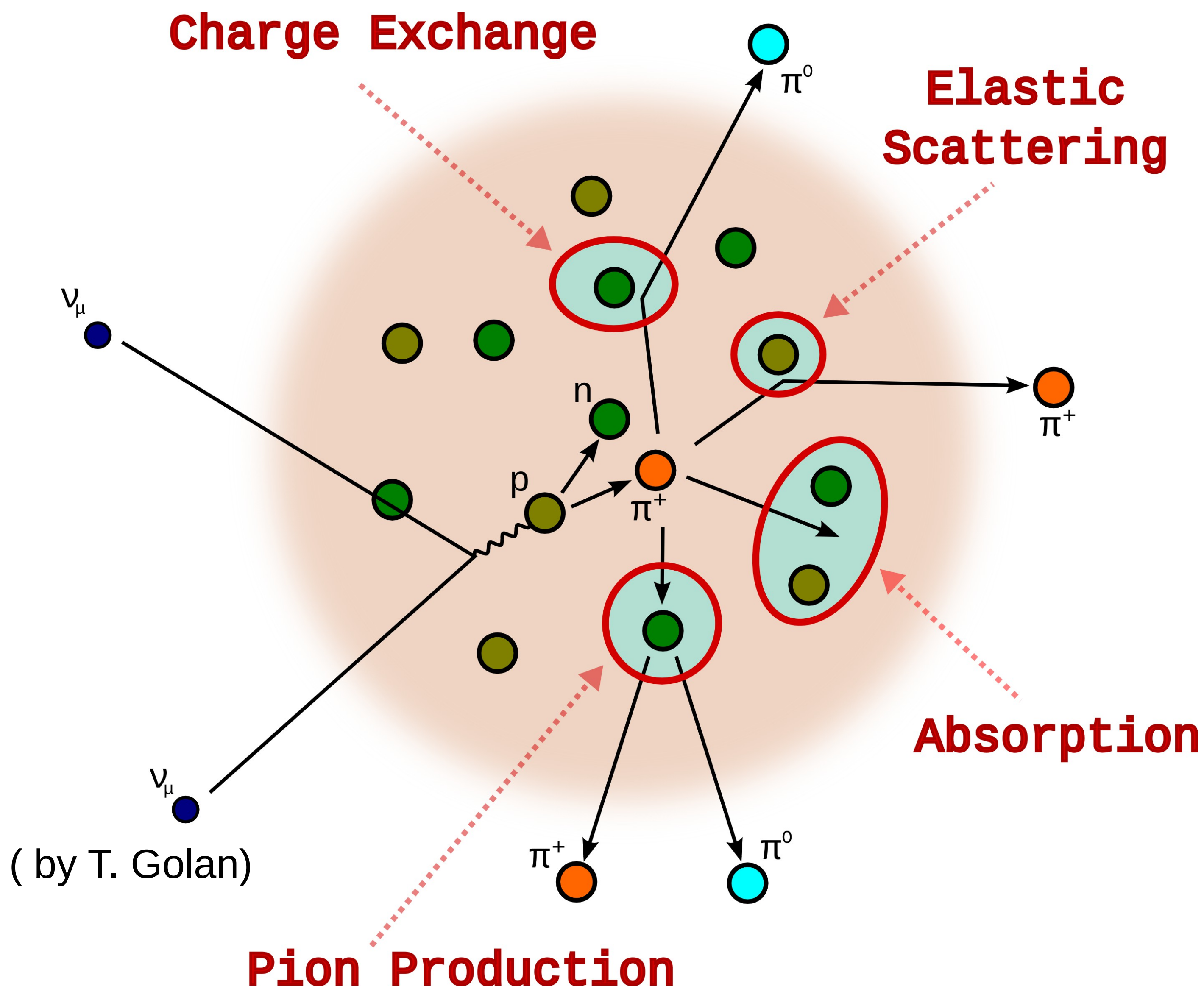


- Next: we use all the mentioned tricks for FSI

All-in-one example: Intranuclear cascade

All-in-one example: intranuclear cascade

- All particles start inside nucleus. Way out: a lot can happen

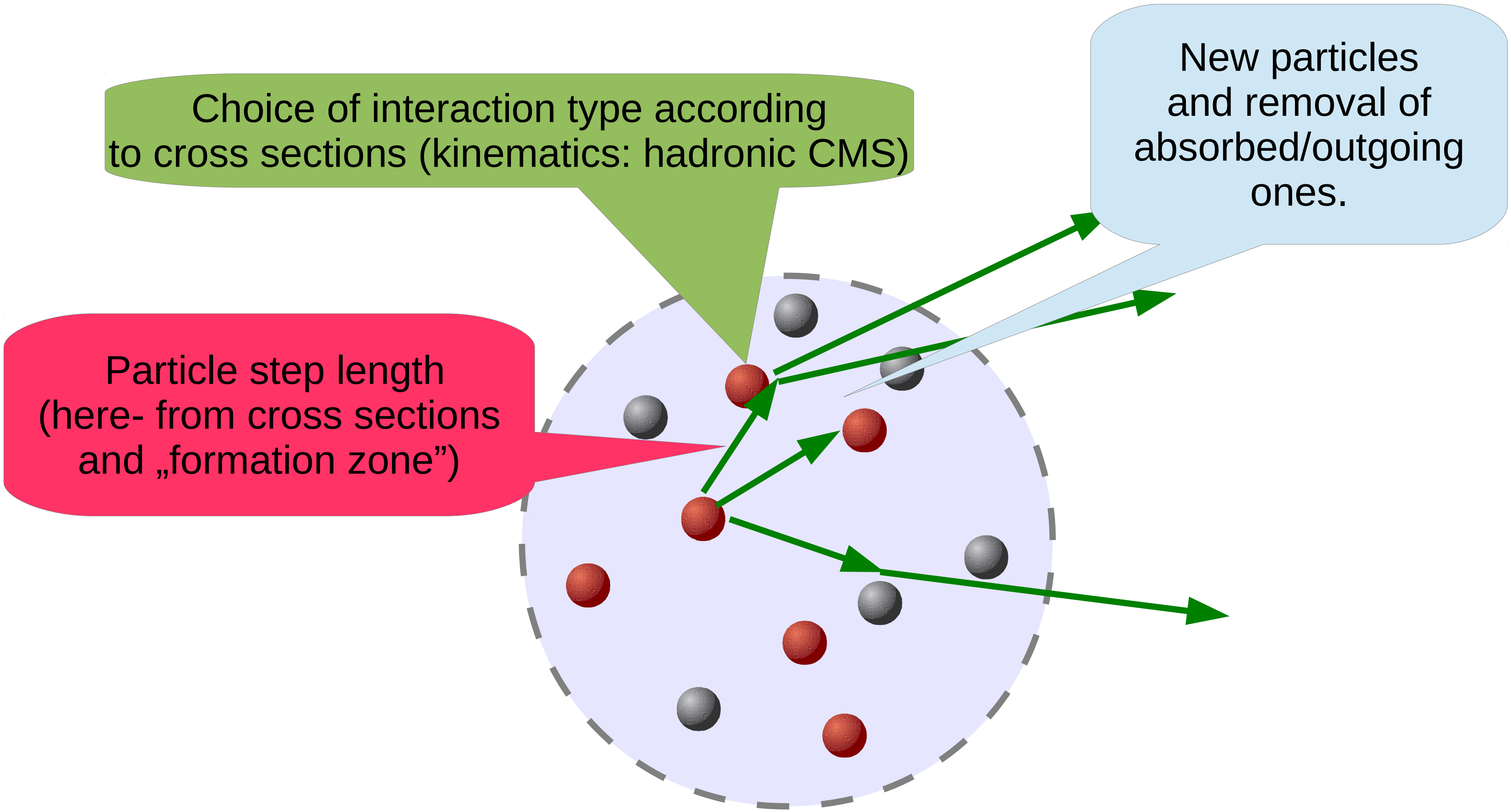


(by T. Golan)

- FSI: quantum transport equations or intranuclear cascade (NuWro).

All-in-one example: intranuclear cascade

1) Handling probabilities, reference frame change, effective modeling → all in one (T. Golan -upcoming PhD thesis!).



All-in-one example: intranuclear cascade – particle step

- Probability of interaction with nucleon at distance *x*:

$$P(\lambda) = \exp(-\lambda/\bar{\lambda})$$

Mean free path: dependence on nuclear matter density ρ and cross σ sections (EL, CEX, PB, ABS, nπ).

- Exponential distribution sampling:

$$\lambda(r) = -\frac{1}{\sigma_p \rho_p(r) + \sigma_n(r)\rho_n(r)} \ln(frand())$$

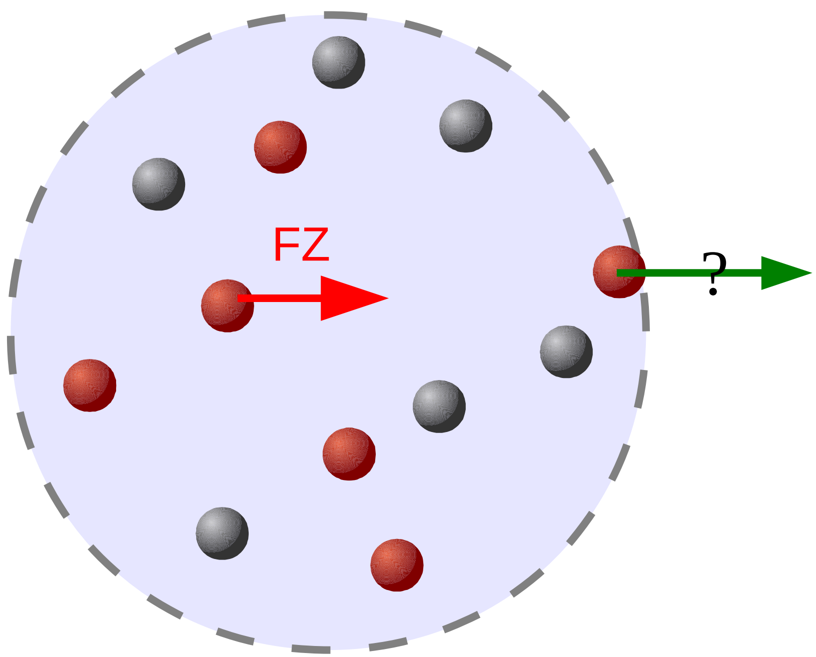
- Propagation by *λ(r)* → sometimes too big w.r.t. typical nuclear matter density changes. Introduction of λ_{max}= e.g. 0.2 fm.
- Cascade step $min[\lambda(r),\lambda_{max}]$. Interaction if $\lambda(r)<\lambda_{max}$.

All-in-one example: intranuclear cascade - particle step

- Example of Formation Zone (FZ) (new particle interaction possible after given distance): first move by the FZ length → different model for nucleons and pions.
- Outside of nucleus condition:

1) Global FG: $r>r_0A^{1/3}$; $r_0=1.25+/-0.20$ fm.

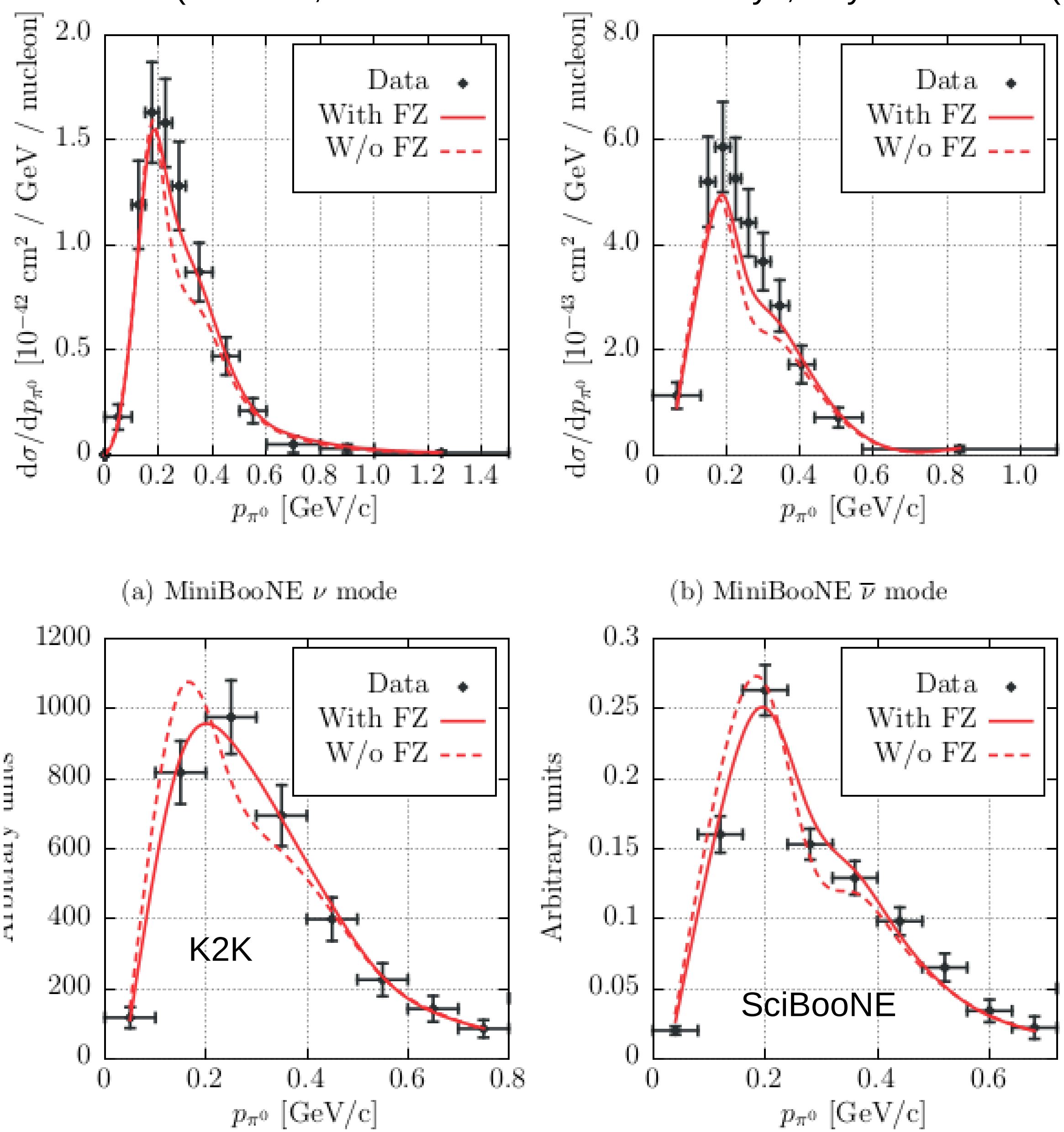
2) Local FG: local densty smaller, than some small fraction of ρ_{max} (e.g. $10^{-6}\rho_{max}$)



All-in-one example: intranuclear cascade - particle step

- Importance of Formation Zone effect

(T. Golan, C. Juszczak and J.T. Sobczyk, Phys. Rev. C86 (2012) 015505)



Better agreement of MC with data!

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All-in-one example: intranuclear cascade - particle step

- Case of Formation Zone (FZ) (new particle interaction possible after given distance): first move by the FZ length → different model for nucleons and pions.
- Cascade step $min[\lambda(r),\lambda_{max}]$. Interaction if $\lambda(r)<\lambda_{max}$.
- Outside of nucleus condition:

1) Global FG: $r>r_0 A^{1/3}$; $r_0=1.25+/-0.20$ *fm*.

2) Local FG: local densty smaller, than some small fraction of ρ_{max} (e.g. $10^{-6}\rho_{max}$)

Tip for nucleon propagation: remember about nucleus potential energy V(r)
and density reduction after removal from nucleus
(proportional local density reduction).
Otherwise
constant density (unlimited nucleon supply)
+
Fermi motion (extra energy for interactions)
=
20+ protons knocked out by 300 MeV neutrino of Carbon nucleus !

All-in-one example: intranuclear cascade – interaction models

- Cross sections for nucleons:

  Metropolis et al. model from Phys.Rev. 105 (1957) 302-310

  plus some corrections and extra points from modern experimental data.

  Storage in data tables in function of nucleon kinetic energy between 350 and 3900 MeV. Below 350 MeV → analytic function of velocity from fit to higher energy data, above: constant values.
- Cross sections for pions: either

  Metropolis et al. (Phys.Rev. 110 (1958) 204-219) experimental data model

  or microscopic calculation (default):

  E. Oset, L.L. Salcedo, D. Strottman, Phys.Lett. B165 (1985) 13-18 → L.L. Salcedo, E. Oset, M.J. Vicente-Vacas, C. Garcia-Recio Nucl.Phys. A484 (1988) 557

All-in-one example: intranuclear cascade – interaction models

- Probabilities in microscopic model ($N\pi^{\lambda} \rightarrow N\pi^{\lambda'}$):

Isospin coefficient matrix

Pauli-blocked Delta width

$$P_{\lambda\lambda'}(k) = M_{\lambda\lambda'}(A,Z)\,P(k)$$

$$P(k) = \frac{2f^{*2}}{3E_\pi m_\pi^2} \int \frac{d^3p}{(2\pi)^3} \rho(\vec{p})\, \vec{k}^2_{CMS} \left| \frac{1}{W - M_\Delta + i\left(\frac{1}{2}\Gamma_\Delta^{PB} - \Im\Sigma_\Delta\right)} \right|^2 \left(\frac{1}{2}\Gamma_\Delta^{PB} - \Im\Sigma_\Delta\right)$$

Nucleon momenta

Delta propagator

Delta self-energy (multinucleon absorption)

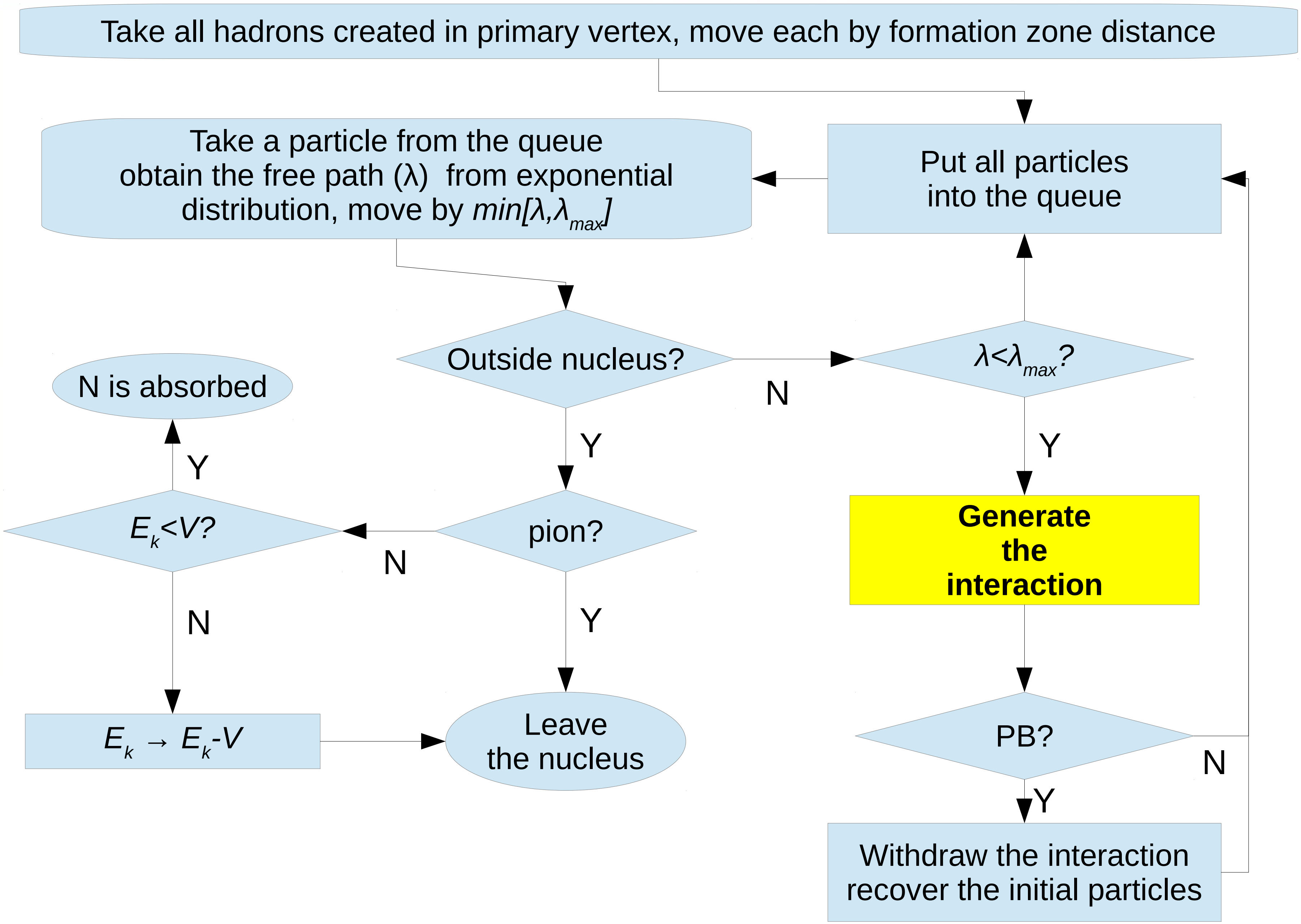
- Delta self- energy calculated and parametrized by E. Oset and L.L. Salcedo, Nucl. Phys. A468 (1987) 631.
- Metropolis-like tables in pion kinetic energy AND nuclear matter density (constant steps for quick search and interpolation).

All-in-one example: intranuclear cascade

- After random interaction choice: random momentum sampling from local Fermi ball, kinematics done in hadronic CMS (symmetries!).
- Path randomization → covered, cross section → covered, kinematics → covered, interaction choice → covered
- Challenge: hadrons go but also new hadrons (one and two-pion production) come.
- Solution: put your hadrons to a queue:

- „Attended” and new hadrons → to the back. Outgoing: remove

All-in-one example: intranuclear cascade



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Summary

- We opened the „black box” of MC:

1) General scheme of neutrino interaction generator algorithm
2) Handling complicated physcal models including MEC and FSI with step-by step algorithms

- Many ways to improve your MC:

1) Choice of probability sampling order (beam-detector-interaction).
2) Choice of sampling routines → fast cummulative distributions from histograms.
3) Weights: possible to compute while running code (test events) → new processes/parameter changes done easy.
4) Troublesome (peaked/ growing with neutrino energy) cross sections: sampling stabilization through re-weighting.
5) As much as possible analytical solutions (e.g. kinematic limits) : better efficiency/speed.
6) Appropriate CMS = higher symmetry → easy phase-space and kinematics.
7) Complicated (time-consuming) cross section computations: choice of minimal information set (e.g. response functions for MEC or probability grid for SF), pre-computation and storage.

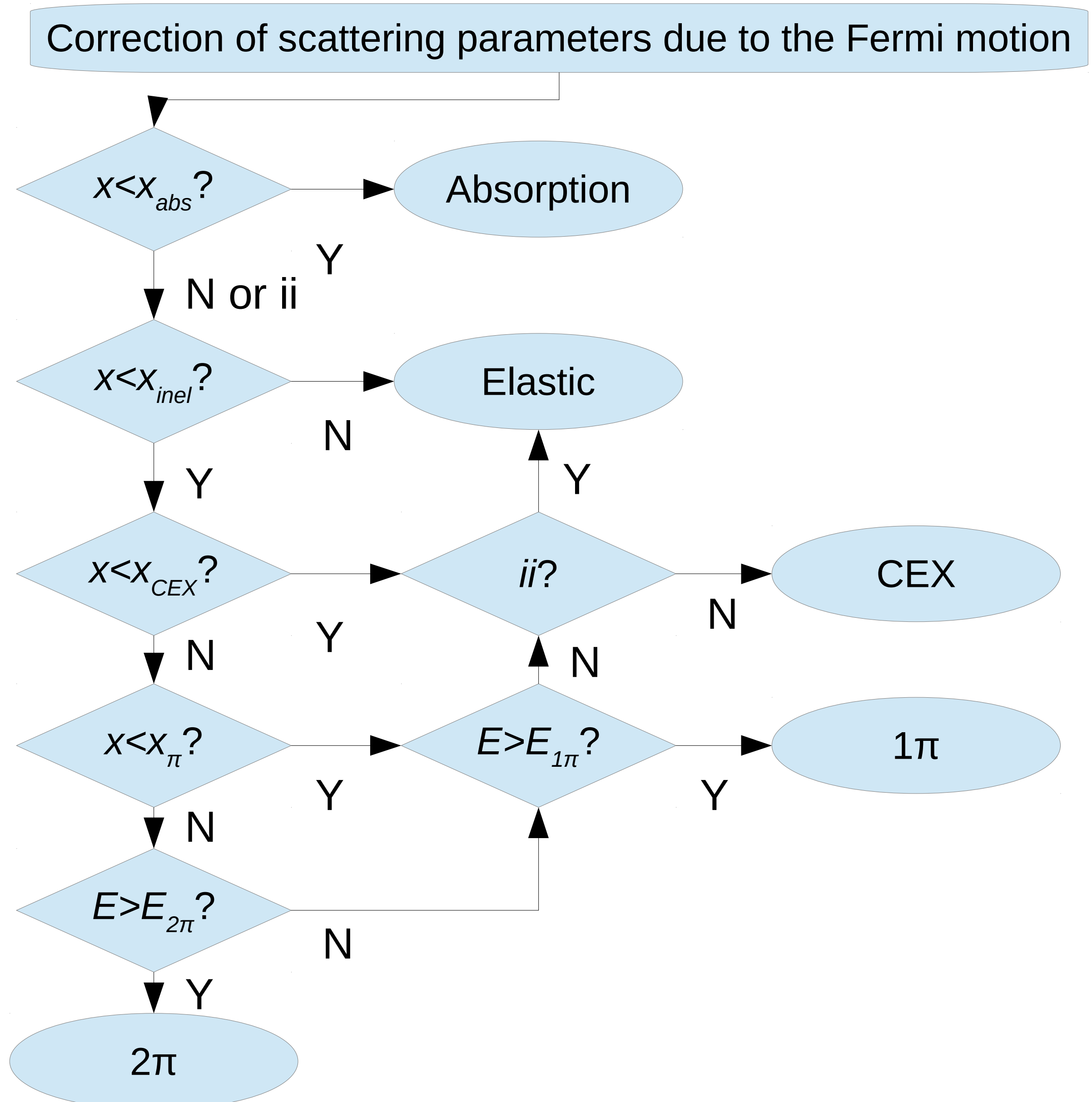
Thank you for your attention!

- Special thanks to T. Golan, K. Graczyk, C. Juszczak J. Sobczyk for discussion and guidance and thanks to T. Golan for giving me access to his thesis. Last, but not least -to the Organizers for wonderful time in Valencia!

Backup

All-in-one example: intranuclear cascade

- „Generate Interaction” for pions (x=frand(), x_i-process probabilities, „ii” → same isospins in target nucleon pair)



All-in-one example: intranuclear cascade

- „Generate Interaction” for nucleons (x=frand(), x_i-process probabilities)